3 Cost-Volume-Profit Analysis

Learning Objectives

1. Explain the features of cost-volume-profit (CVP) analysis
2. Determine the breakeven point and output level needed to achieve a target operating income
3. Understand how income taxes affect CVP analysis
4. Explain how managers use CVP analysis in decision making
5. Explain how sensitivity analysis helps managers cope with uncertainty
6. Use CVP analysis to plan variable and fixed costs
7. Apply CVP analysis to a company producing multiple products

All managers want to know how profits will change as the units sold of a product or service change.

Home Depot managers, for example, might wonder how many units of a new product must be sold to break even or make a certain amount of profit. Procter & Gamble managers might ask themselves how expanding their business into a particular foreign market would affect costs, selling price, and profits. These questions have a common "what-if" theme. Examining the results of these what-if possibilities and alternatives helps managers make better decisions.

Managers must also decide how to price their products and understand the effect of their pricing decisions on revenues and profits. The following article explains how the Irish rock band U2 recently decided whether it should decrease the prices on some of its tickets during its recent world tour. Does lowering ticket price sound like a wise strategy to you?

How the "The Biggest Rock Show Ever" Turned a Big Profit[1]

When U2 embarked on its recent world tour, *Rolling Stone* magazine called it "the biggest rock show ever." Visiting large stadiums across the United States and Europe, the Irish quartet performed on an imposing 164-foot high stage that resembled a spaceship, complete with a massive video screen and footbridges leading to ringed catwalks.

With an ambitious 48-date trek planned, U2 actually had three separate stages leapfrogging its global itinerary—each one costing nearly $40 million dollars. As a result, the tour's success was dependent not only on each night's concert, but also recouping its tremendous fixed costs—costs that do not change with the number of fans in the audience.

To cover its high fixed costs and make a profit, U2 needed to sell a lot of tickets. To maximize revenue, the tour employed a unique in-the-round stage configuration, which boosted stadium capacity by roughly 20%, and sold tickets for as little as $30, far less than most large outdoor concerts.

The band's plan worked—despite a broader music industry slump and global recession, U2 shattered attendance records in most of the venues it played. By the end of the tour, the band played to over

[1] *Source*: Gundersen, Edna. 2009. U2 turns 360 stadium into attendance-shattering sellouts. *USA Today*, October 4. *www.usatoday.com/life/music/news/2009-10-04-u2-stadium-tour_N.htm*

3 million fans, racking up almost $300 million in ticket and merchandise sales and turning a profit. As you read this chapter, you will begin to understand how and why U2 made the decision to lower prices.

Many capital intensive companies, such as US Airways and United Airlines in the airlines industry and Global Crossing and WorldCom in the telecommunications industry, have high fixed costs. They must generate sufficient revenues to cover these costs and turn a profit. When revenues declined at these companies during 2001 and 2002 and fixed costs remained high, these companies declared bankruptcy. The methods of CVP analysis described in this chapter help managers minimize such risks.

Essentials of CVP Analysis

Learning Objective 1

Explain the features of cost-volume-profit (CVP) analysis

. . . how operating income changes with changes in output level, selling prices, variable costs, or fixed costs

In Chapter 2, we discussed total revenues, total costs, and income. **Cost-volume-profit (CVP) analysis** studies the behavior and relationship among these elements as changes occur in the units sold, the selling price, the variable cost per unit, or the fixed costs of a product. Let's consider an example to illustrate CVP analysis.

> Example: Emma Frost is considering selling GMAT Success, a test prep book and software package for the business school admission test, at a college fair in Chicago. Emma knows she can purchase this package from a wholesaler at $120 per package, with the privilege of returning all unsold packages and receiving a full $120 refund per package. She also knows that she must pay $2,000 to the organizers for the booth rental at the fair. She will incur no other costs. She must decide whether she should rent a booth.

Emma, like most managers who face such a situation, works through a series of steps.

1. **Identify the problem and uncertainties.** The decision to rent the booth hinges critically on how Emma resolves two important uncertainties—the price she can charge and the number of packages she can sell at that price. Every decision deals with selecting a course of action. Emma must decide knowing that the outcome of the chosen action is uncertain and will only be known in the future. The more confident Emma is about selling a large number of packages at a good price, the more willing she will be to rent the booth.
2. **Obtain information.** When faced with uncertainty, managers obtain information that might help them understand the uncertainties better. For example, Emma gathers information about the type of individuals likely to attend the fair and other test-prep packages that might be sold at the fair. She also gathers data on her past experiences selling GMAT Success at fairs very much like the Chicago fair.

3. **Make predictions about the future.** Using all the information available to them, managers make predictions. Emma predicts that she can charge a price of $200 for GMAT Success. At that price she is reasonably confident that she will be able to sell at least 30 packages and possibly as many as 60. In making these predictions, Emma like most managers, must be realistic and exercise careful judgment. If her predictions are excessively optimistic, Emma will rent the booth when she should not. If they are unduly pessimistic, Emma will not rent the booth when she should.

   Emma's predictions rest on the belief that her experience at the Chicago fair will be similar to her experience at the Boston fair four months earlier. Yet, Emma is uncertain about several aspects of her prediction. Is the comparison between Boston and Chicago appropriate? Have conditions and circumstances changed over the last four months? Are there any biases creeping into her thinking? She is keen on selling at the Chicago fair because sales in the last couple of months have been lower than expected. Is this experience making her predictions overly optimistic? Has she ignored some of the competitive risks? Will the other test prep vendors at the fair reduce their prices?

   Emma reviews her thinking. She retests her assumptions. She also explores these questions with John Mills, a close friend, who has extensive experience selling test-prep packages like GMAT Success. In the end, she feels quite confident that her predictions are reasonable, accurate, and carefully thought through.
4. **Make decisions by choosing among alternatives.** Emma uses the CVP analysis that follows, and decides to rent the booth at the Chicago fair.
5. **Implement the decision, evaluate performance, and learn.** Thoughtful managers never stop learning. They compare their actual performance to predicted performance to understand why things worked out the way they did and what they might learn. At the end of the Chicago fair, for example, Emma would want to evaluate whether her predictions about price and the number of packages she could sell were correct. Such feedback would be very helpful to Emma as she makes decisions about renting booths at subsequent fairs.

How does Emma use CVP analysis in Step 4 to make her decision? Emma begins by identifying which costs are fixed and which costs are variable and then calculates *contribution margin*.

Contribution Margins

The booth-rental cost of $2,000 is a fixed cost because it will not change no matter how many packages Emma sells. The cost of the package itself is a variable cost because it increases in proportion to the number of packages sold. Emma will incur a cost of $120 for each package that she sells. To get an idea of how operating income will change as a result of selling different quantities of packages, Emma calculates operating income if sales are 5 packages and if sales are 40 packages.

	5 packages sold		40 packages sold	
Revenues	$ 1,000	($200 per package × 5 packages)	$8,000	($200 per package × 40 packages)
Variable purchase costs	600	($120 per package × 5 packages)	4,800	($120 per package × 40 packages)
Fixed costs	2,000		2,000	
Operating income	$(1,600)		$1,200	

The only numbers that change from selling different quantities of packages are *total revenues* and *total variable costs*. The difference between total revenues and total variable costs is called **contribution margin**. That is,

$$\text{Contribution margin} = \text{Total revenues} - \text{Total variable costs}$$

Contribution margin indicates why operating income changes as the number of units sold changes. The contribution margin when Emma sells 5 packages is $400 ($1,000 in total revenues minus $600 in total variable costs); the contribution margin when Emma sells

40 packages is $3,200 ($8,000 in total revenues minus $4,800 in total variable costs). When calculating the contribution margin, be sure to subtract all variable costs. For example, if Emma had variable selling costs because she paid a commission to salespeople for each package they sold at the fair, variable costs would include the cost of each package plus the sales commission.

Contribution margin per unit is a useful tool for calculating contribution margin and operating income. It is defined as,

$$\text{Contribution margin per unit} = \text{Selling price} - \text{Variable cost per unit}$$

In the GMAT Success example, contribution margin per package, or per unit, is $200 − $120 = $80. Contribution margin per unit recognizes the tight coupling of selling price and variable cost per unit. Unlike fixed costs, Emma will only incur the variable cost per unit of $120 when she sells a unit of GMAT Success for $200.

Contribution margin per unit provides a second way to calculate contribution margin:

$$\text{Contribution margin} = \text{Contribution margin per unit} \times \text{Number of units sold}$$

For example, when 40 packages are sold, contribution margin = $80 per unit × 40 units = $3,200.

Even before she gets to the fair, Emma incurs $2,000 in fixed costs. Because the contribution margin per unit is $80, Emma will recover $80 for each package that she sells at the fair. Emma hopes to sell enough packages to fully recover the $2,000 she spent for renting the booth and to then start making a profit.

Exhibit 3-1 presents contribution margins for different quantities of packages sold. The income statement in Exhibit 3-1 is called a **contribution income statement** because it groups costs into variable costs and fixed costs to highlight contribution margin. Each additional package sold from 0 to 1 to 5 increases contribution margin by $80 per package, recovering more of the fixed costs and reducing the operating loss. If Emma sells 25 packages, contribution margin equals $2,000 ($80 per package × 25 packages), exactly recovering fixed costs and resulting in $0 operating income. If Emma sells 40 packages, contribution margin increases by another $1,200 ($3,200 − $2,000), all of which becomes operating income. As you look across Exhibit 3-1 from left to right, you see that the increase in contribution margin exactly equals the increase in operating income (or the decrease in operating loss).

Instead of expressing contribution margin as a dollar amount per unit, we can express it as a percentage called **contribution margin percentage** (or **contribution margin ratio**):

$$\text{Contribution margin percentage (or contribution margin ratio)} = \frac{\text{Contribution margin per unit}}{\text{Selling price}}$$

In our example,

$$\text{Contribution margin percentage} = \frac{\$80}{\$200} = 0.40, \text{ or } 40\%$$

Contribution margin percentage is the contribution margin per dollar of revenue. Emma earns 40% of each dollar of revenue (equal to 40 cents).

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	A	B	C	D	E	F	G	H
1				Number of Packages Sold				
2				0	1	5	25	40
3	Revenues	$ 200	per package	$ 0	$ 200	$ 1,000	$5,000	$8,000
4	Variable costs	$ 120	per package	0	120	600	3,000	4,800
5	Contribution margin	$ 80	per package	0	80	400	2,000	3,200
6	Fixed costs	$2,000		2,000	2,000	2,000	2,000	2,000
7	Operating income			$(2,000)	$(1,920)	$(1,600)	$ 0	$1,200

Exhibit 3-1

Contribution Income Statement for Different Quantities of GMAT Success Packages Sold

Most companies have multiple products. As we shall see later in this chapter, calculating contribution margin per unit when there are multiple products is more cumbersome. In practice, companies routinely use contribution margin percentage as a handy way to calculate contribution margin for different dollar amounts of revenue:

$$\text{Contribution margin} = \text{Contribution margin percentage} \times \text{Revenues (in dollars)}$$

For example, in Exhibit 3-1, if Emma sells 40 packages, revenues will be \$8,000 and contribution margin will equal 40% of \$8,000, or $0.40 \times \$8{,}000 = \$3{,}200$. Emma earns operating income of \$1,200 (\$3,200 − Fixed costs, \$2,000) by selling 40 packages for \$8,000.

Expressing CVP Relationships

How was the Excel spreadsheet in Exhibit 3-1 constructed? Underlying the Exhibit are some equations that express the CVP relationships. To make good decisions using CVP analysis, we must understand these relationships and the structure of the contribution income statement in Exhibit 3-1. There are three related ways (we will call them methods) to think more deeply about and model CVP relationships:

1. The equation method
2. The contribution margin method
3. The graph method

The equation method and the contribution margin method are most useful when managers want to determine operating income at few specific levels of sales (for example 5, 15, 25, and 40 units sold). The graph method helps managers visualize the relationship between units sold and operating income over a wide range of quantities of units sold. As we shall see later in the chapter, different methods are useful for different decisions.

Equation Method

Each column in Exhibit 3-1 is expressed as an equation.

$$\text{Revenues} - \text{Variable costs} - \text{Fixed costs} = \text{Operating income}$$

How are revenues in each column calculated?

$$\text{Revenues} = \text{Selling price } (SP) \times \text{Quantity of units sold } (Q)$$

How are variable costs in each column calculated?

$$\text{Variable costs} = \text{Variable cost per unit } (VCU) \times \text{Quantity of units sold } (Q)$$

So,

$$\left[\left(\begin{matrix}\text{Selling}\\\text{price}\end{matrix} \times \begin{matrix}\text{Quantity of}\\\text{units sold}\end{matrix}\right) - \left(\begin{matrix}\text{Variable cost}\\\text{per unit}\end{matrix} \times \begin{matrix}\text{Quantity of}\\\text{units sold}\end{matrix}\right)\right] - \begin{matrix}\text{Fixed}\\\text{costs}\end{matrix} = \begin{matrix}\text{Operating}\\\text{income}\end{matrix} \qquad \textbf{(Equation 1)}$$

Equation 1 becomes the basis for calculating operating income for different quantities of units sold. For example, if you go to cell F7 in Exhibit 3-1, the calculation of operating income when Emma sells 5 packages is

$$(\$200 \times 5) - (\$120 \times 5) - \$2{,}000 = \$1{,}000 - \$600 - \$2{,}000 = -\$1{,}600$$

Contribution Margin Method

Rearranging equation 1,

$$\left[\left(\begin{matrix}\text{Selling}\\\text{price}\end{matrix} - \begin{matrix}\text{Variable cost}\\\text{per unit}\end{matrix}\right) \times \left(\begin{matrix}\text{Quantity of}\\\text{units sold}\end{matrix}\right)\right] - \begin{matrix}\text{Fixed}\\\text{costs}\end{matrix} = \begin{matrix}\text{Operating}\\\text{income}\end{matrix}$$

$$\left(\begin{matrix}\text{Contribution margin}\\\text{per unit}\end{matrix} \times \begin{matrix}\text{Quantity of}\\\text{units sold}\end{matrix}\right) - \begin{matrix}\text{Fixed}\\\text{costs}\end{matrix} = \begin{matrix}\text{Operating}\\\text{income}\end{matrix} \qquad \textbf{(Equation 2)}$$

In our GMAT Success example, contribution margin per unit is \$80 (\$200 − \$120), so when Emma sells 5 packages,

$$\text{Operating income} = (\$80 \times 5) - \$2{,}000 = -\$1{,}600$$

Equation 2 expresses the basic idea we described earlier—each unit sold helps Emma recover \$80 (in contribution margin) of the \$2,000 in fixed costs.

Graph Method

In the graph method, we represent total costs and total revenues graphically. Each is shown as a line on a graph. Exhibit 3-2 illustrates the graph method for GMAT Success. Because we have assumed that total costs and total revenues behave in a linear fashion, we need only two points to plot the line representing each of them.

1. **Total costs line.** The total costs line is the sum of fixed costs and variable costs. Fixed costs are \$2,000 for all quantities of units sold within the relevant range. To plot the total costs line, use as one point the \$2,000 fixed costs at zero units sold (point A) because variable costs are \$0 when no units are sold. Select a second point by choosing any other convenient output level (say, 40 units sold) and determine the corresponding total costs. Total variable costs at this output level are \$4,800 (40 units × \$120 per unit). Remember, fixed costs are \$2,000 at all quantities of units sold within the relevant range, so total costs at 40 units sold equal \$6,800 (\$2,000 + \$4,800), which is point B in Exhibit 3-2. The total costs line is the straight line from point A through point B.
2. **Total revenues line.** One convenient starting point is \$0 revenues at 0 units sold, which is point C in Exhibit 3-2. Select a second point by choosing any other convenient output level and determining the corresponding total revenues. At 40 units sold, total revenues are \$8,000 (\$200 per unit × 40 units), which is point D in Exhibit 3-2. The total revenues line is the straight line from point C through point D.

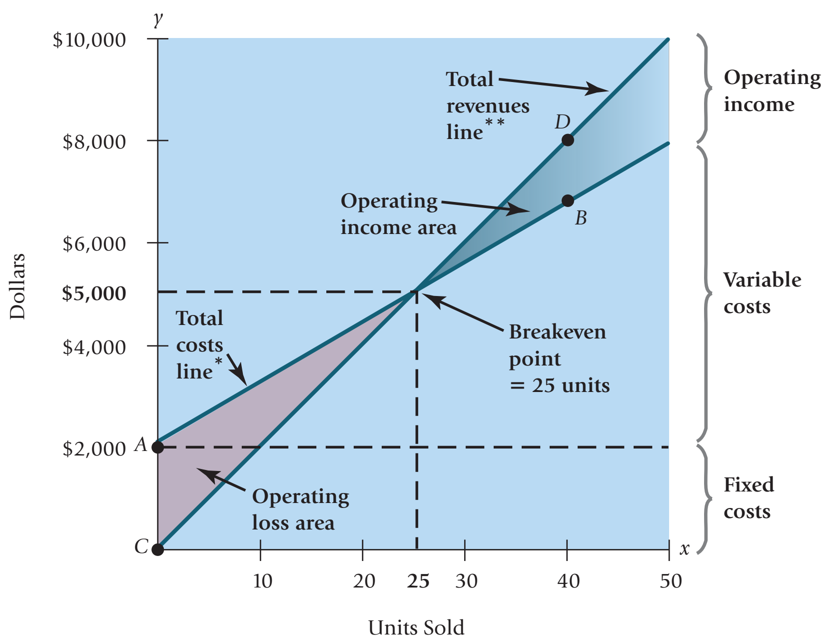
   Profit or loss at any sales level can be determined by the vertical distance between the two lines at that level in Exhibit 3-2. For quantities fewer than 25 units sold, total costs exceed total revenues, and the purple area indicates operating losses. For quantities greater than 25 units sold, total revenues exceed total costs, and the blue-green area indicates operating incomes. At 25 units sold, total revenues equal total costs. Emma will break even by selling 25 packages.

Decision Point

How can CVP analysis assist managers?

Exhibit 3-2

Cost-Volume Graph for GMAT Success



*Slope of the total costs line is the variable cost per unit = \$120
**Slope of the total revenues line is the selling price = \$200

Cost-Volume-Profit Assumptions

Now that you have seen how CVP analysis works, think about the following assumptions we made during the analysis:

1. Changes in the levels of revenues and costs arise only because of changes in the number of product (or service) units sold. The number of units sold is the only revenue driver and the only cost driver. Just as a cost driver is any factor that affects costs, a **revenue driver** is a variable, such as volume, that causally affects revenues.
2. Total costs can be separated into two components: a fixed component that does not vary with units sold and a variable component that changes with respect to units sold.
3. When represented graphically, the behaviors of total revenues and total costs are linear (meaning they can be represented as a straight line) in relation to units sold within a relevant range (and time period).
4. Selling price, variable cost per unit, and total fixed costs (within a relevant range and time period) are known and constant.

As the CVP assumptions make clear, an important feature of CVP analysis is distinguishing fixed from variable costs. Always keep in mind, however, that whether a cost is variable or fixed depends on the time period for a decision.

The shorter the time horizon, the higher the percentage of total costs considered fixed. For example, suppose an American Airlines plane will depart from its gate in the next hour and currently has 20 seats unsold. A potential passenger arrives with a transferable ticket from a competing airline. The variable costs (such as one more meal) to American of placing one more passenger in an otherwise empty seat is negligible At the time of this decision, with only an hour to go before the flight departs, virtually all costs (such as crew costs and baggage-handling costs) are fixed.

Alternatively, suppose American Airlines must decide whether to keep this flight in its flight schedule. This decision will have a one-year planning horizon. If American Airlines decides to cancel this flight because very few passengers during the last year have taken this flight, many more costs, including crew costs, baggage-handling costs, and airport fees, would be considered variable. That's because over this longer horizon, these costs would not have to be incurred if the flight were no longer operating. Always consider the relevant range, the length of the time horizon, and the specific decision situation when classifying costs as variable or fixed.

Breakeven Point and Target Operating Income

Learning Objective 2

Determine the breakeven point and output level needed to achieve a target operating income

. . . compare contribution margin and fixed costs

Managers and entrepreneurs like Emma always want to know how much they must sell to earn a given amount of income. Equally important, they want to know how much they must sell to avoid a loss.

Breakeven Point

The **breakeven point (BEP)** is that quantity of output sold at which total revenues equal total costs—that is, the quantity of output sold that results in $0 of operating income. We have already seen how to use the graph method to calculate the breakeven point. Recall from Exhibit 3-1 that operating income was $0 when Emma sold 25 units, the breakeven point. But by understanding the equations underlying the calculations in Exhibit 3-1, we can calculate the breakeven point directly for GMAT Success rather than trying out different quantities and checking when operating income equals $0.

Recall the equation method (equation 1):

$$\left(\begin{array}{c}\text{Selling}\\\text{price}\end{array} \times \begin{array}{c}\text{Quantity of}\\\text{units sold}\end{array}\right) - \left(\begin{array}{c}\text{Variable cost}\\\text{per unit}\end{array} \times \begin{array}{c}\text{Quantity of}\\\text{units sold}\end{array}\right) - \begin{array}{c}\text{Fixed}\\\text{costs}\end{array} = \begin{array}{c}\text{Operating}\\\text{income}\end{array}$$

Setting operating income equal to \$0 and denoting quantity of output units that must be sold by Q,

$$(\$200 \times Q) - (\$120 \times Q) - \$2{,}000 = \$0$$
$$\$80 \times Q = \$2{,}000$$
$$Q = \$2{,}000 \div \$80 \text{ per unit} = 25 \text{ units}$$

If Emma sells fewer than 25 units, she will incur a loss; if she sells 25 units, she will break even; and if she sells more than 25 units, she will make a profit. While this breakeven point is expressed in units, it can also be expressed in revenues: 25 units × \$200 selling price = \$5,000.

Recall the contribution margin method (equation 2):

$$\left(\begin{array}{c}\text{Contribution}\\ \text{margin per unit}\end{array} \times \begin{array}{c}\text{Quantity of}\\ \text{units sold}\end{array}\right) - \text{Fixed costs} = \text{Operating income}$$

At the breakeven point, operating income is by definition \$0 and so,

$$\text{Contribution margin per unit} \times \text{Breakeven number of units} = \text{Fixed cost} \qquad \textbf{(Equation 3)}$$

Rearranging equation 3 and entering the data,

$$\begin{array}{c}\text{Breakeven}\\ \text{number of units}\end{array} = \frac{\text{Fixed costs}}{\text{Contribution margin per unit}} = \frac{\$2{,}000}{\$80 \text{ per unit}} = 25 \text{ units}$$

$$\text{Breakeven revenues} = \text{Breakeven number of units} \times \text{Selling price}$$
$$= 25 \text{ units} \times \$200 \text{ per unit} = \$5{,}000$$

In practice (because they have multiple products), companies usually calculate breakeven point directly in terms of revenues using contribution margin percentages. Recall that in the GMAT Success example,

$$\begin{array}{c}\text{Contribution margin}\\ \text{percentage}\end{array} = \frac{\text{Contribution margin per unit}}{\text{Selling price}} = \frac{\$80}{\$200} = 0.40, \text{ or } 40\%$$

That is, 40% of each dollar of revenue, or 40 cents, is contribution margin. To break even, contribution margin must equal fixed costs of \$2,000. To earn \$2,000 of contribution margin, when \$1 of revenue earns \$0.40 of contribution margin, revenues must equal \$2,000 ÷ 0.40 = \$5,000.

$$\begin{array}{c}\text{Breakeven}\\ \text{revenues}\end{array} = \frac{\text{Fixed costs}}{\text{Contribution margin \%}} = \frac{\$2{,}000}{0.40} = \$5{,}000$$

While the breakeven point tells managers how much they must sell to avoid a loss, managers are equally interested in how they will achieve the operating income targets underlying their strategies and plans. In our example, selling 25 units at a price of \$200 assures Emma that she will not lose money if she rents the booth. This news is comforting, but we next describe how Emma determines how much she needs to sell to achieve a targeted amount of operating income.

Target Operating Income

We illustrate target operating income calculations by asking the following question: How many units must Emma sell to earn an operating income of \$1,200? One approach is to keep plugging in different quantities into Exhibit 3-1 and check when operating income equals \$1,200. Exhibit 3-1 shows that operating income is \$1,200 when 40 packages are sold. A more convenient approach is to use equation 1 from page 66.

$$\left[\left(\begin{array}{c}\text{Selling}\\ \text{price}\end{array} \times \begin{array}{c}\text{Quantity of}\\ \text{units sold}\end{array}\right) - \left(\begin{array}{c}\text{Variable cost}\\ \text{per unit}\end{array} \times \begin{array}{c}\text{Quantity of}\\ \text{units sold}\end{array}\right)\right] - \begin{array}{c}\text{Fixed}\\ \text{costs}\end{array} = \begin{array}{c}\text{Operating}\\ \text{income}\end{array} \qquad \textbf{(Equation 1)}$$

We denote by Q the unknown quantity of units Emma must sell to earn an operating income of \$1,200. Selling price is \$200, variable cost per package is \$120, fixed costs are

$2,000, and target operating income is $1,200. Substituting these values into equation 1, we have

$$(\$200 \times Q) - (\$120 \times Q) - \$2{,}000 = \$1{,}200$$
$$\$80 \times Q = \$2{,}000 + \$1{,}200 = \$3{,}200$$
$$Q = \$3{,}200 \div \$80 \text{ per unit} = 40 \text{ units}$$

Alternatively, we could use equation 2,

$$\left(\begin{array}{c}\text{Contribution margin}\\ \text{per unit}\end{array} \times \begin{array}{c}\text{Quantity of}\\ \text{units sold}\end{array}\right) - \begin{array}{c}\text{Fixed}\\ \text{costs}\end{array} = \begin{array}{c}\text{Operating}\\ \text{income}\end{array} \qquad \textbf{(Equation 2)}$$

Given a target operating income ($1,200 in this case), we can rearrange terms to get equation 4.

$$\begin{array}{c}\text{Quantity of units}\\ \text{required to be sold}\end{array} = \frac{\text{Fixed costs} + \text{Target operating income}}{\text{Contribution margin per unit}} \qquad \textbf{(Equation 4)}$$

$$\begin{array}{c}\text{Quantity of units}\\ \text{required to be sold}\end{array} = \frac{\$2{,}000 + \$1{,}200}{\$80 \text{ per unit}} = 40 \text{ units}$$

Proof:	Revenues, $200 per unit × 40 units	$8,000
	Variable costs, $120 per unit × 40 units	4,800
	Contribution margin, $80 per unit × 40 units	3,200
	Fixed costs	2,000
	Operating income	$1,200

The revenues needed to earn an operating income of $1,200 can also be calculated directly by recognizing (1) that $3,200 of contribution margin must be earned (fixed costs of $2,000 plus operating income of $1,200) and (2) that $1 of revenue earns $0.40 (40 cents) of contribution margin. To earn $3,200 of contribution margin, revenues must equal $3,200 ÷ 0.40 = $8,000.

$$\text{Revenues needed to earn operating income of \$1,200} = \frac{\$2{,}000 + \$1{,}200}{0.40} = \frac{\$3{,}200}{0.40} = \$8{,}000$$

The graph in Exhibit 3-2 is very difficult to use to answer the question: How many units must Emma sell to earn an operating income of $1,200? Why? Because it is not easy to determine from the graph the precise point at which the difference between the total revenues line and the total costs line equals $1,200. However, recasting Exhibit 3-2 in the form of a profit-volume (PV) graph makes it easier to answer this question.

Decision Point

How can managers determine the breakeven point or the output needed to achieve a target operating income?

A **PV graph** shows how changes in the quantity of units sold affect operating income. Exhibit 3-3 is the PV graph for GMAT Success (fixed costs, $2,000; selling price, $200; and variable cost per unit, $120). The PV line can be drawn using two points. One convenient point (M) is the operating loss at 0 units sold, which is equal to the fixed costs of $2,000, shown at –$2,000 on the vertical axis. A second convenient point (N) is the breakeven point, which is 25 units in our example (see p. 69). The PV line is the straight line from point M through point N. To find the number of units Emma must sell to earn an operating income of $1,200, draw a horizontal line parallel to the *x*-axis corresponding to $1,200 on the vertical axis (that's the *y*-axis). At the point where this line intersects the PV line, draw a vertical line down to the horizontal axis (that's the *x*-axis). The vertical line intersects the *x*-axis at 40 units, indicating that by selling 40 units Emma will earn an operating income of $1,200.

Target Net Income and Income Taxes

Learning Objective 3

Understand how income taxes affect CVP analysis

. . . focus on net income

Net income is operating income plus nonoperating revenues (such as interest revenue) minus nonoperating costs (such as interest cost) minus income taxes. For simplicity, throughout this chapter we assume nonoperating revenues and nonoperating costs are zero. Thus,

$$\text{Net income} = \text{Operating income} - \text{Income taxes}$$

Until now, we have ignored the effect of income taxes in our CVP analysis. In many companies, the income targets for managers in their strategic plans are expressed in terms of

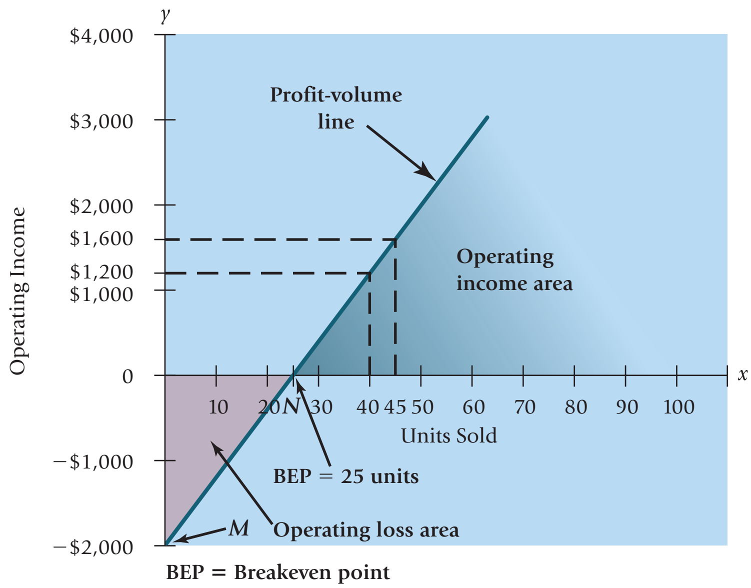


Exhibit 3-3

Profit-Volume Graph for GMAT Success

net income. That's because top management wants subordinate managers to take into account the effects their decisions have on operating income after income taxes. Some decisions may not result in large operating incomes, but they may have favorable tax consequences, making them attractive on a net income basis—the measure that drives shareholders' dividends and returns.

To make net income evaluations, CVP calculations for target income must be stated in terms of target net income instead of target operating income. For example, Emma may be interested in knowing the quantity of units she must sell to earn a net income of $960, assuming an income tax rate of 40%.

$$\text{Target net income} = \left(\begin{matrix}\text{Target}\\\text{operating income}\end{matrix}\right) - \left(\begin{matrix}\text{Target}\\\text{operating income}\end{matrix} \times \text{Tax rate}\right)$$

$$\text{Target net income} = (\text{Target operating income}) \times (1 - \text{Tax rate})$$

$$\text{Target operating income} = \frac{\text{Target net income}}{1 - \text{Tax rate}} = \frac{\$960}{1 - 0.40} = \$1,600$$

In other words, to earn a target net income of $960, Emma's target operating income is $1,600.

Proof: Target operating income		$1,600
Tax at 40% (0.40 × $1,600)		640
Target net income		$ 960

The key step is to take the target net income number and convert it into the corresponding target operating income number. We can then use equation 1 for target operating income and substitute numbers from our GMAT Success example.

$$\left[\left(\begin{matrix}\text{Selling}\\\text{price}\end{matrix} \times \begin{matrix}\text{Quantity of}\\\text{units sold}\end{matrix}\right) - \left(\begin{matrix}\text{Variable cost}\\\text{per unit}\end{matrix} \times \begin{matrix}\text{Quantity of}\\\text{units sold}\end{matrix}\right)\right] - \begin{matrix}\text{Fixed}\\\text{costs}\end{matrix} = \begin{matrix}\text{Operating}\\\text{income}\end{matrix} \qquad \text{(Equation 1)}$$

$$(\$200 \times Q) - (\$120 \times Q) - \$2,000 = \$1,600$$

$$\$80 \times Q = \$3,600$$

$$Q = \$3,600 \div \$80 \text{ per unit} = 45 \text{ units}$$

Alternatively we can calculate the number of units Emma must sell by using the contribution margin method and equation 4:

$$\begin{matrix}\text{Quantity of units}\\\text{required to be sold}\end{matrix} = \frac{\text{Fixed costs} + \text{Target operating income}}{\text{Contribution margin per unit}} \qquad \text{(Equation 4)}$$

$$= \frac{\$2,000 + \$1,600}{\$80 \text{ per unit}} = 45 \text{ units}$$

Proof:	Revenues, $200 per unit × 45 units	$9,000
	Variable costs, $120 per unit × 45 units	5,400
	Contribution margin	3,600
	Fixed costs	2,000
	Operating income	1,600
	Income taxes, $1,600 × 0.40	640
	Net income	$ 960

Decision Point

How can managers incorporate income taxes into CVP analysis?

Emma can also use the PV graph in Exhibit 3-3. To earn target operating income of $1,600, Emma needs to sell 45 units.

Focusing the analysis on target net income instead of target operating income will not change the breakeven point. That's because, by definition, operating income at the breakeven point is $0, and no income taxes are paid when there is no operating income.

Using CVP Analysis for Decision Making

Learning Objective 4

Explain how managers use CVP analysis in decision making

. . . choose the alternative that maximizes operating income

We have seen how CVP analysis is useful for calculating the units that need to be sold to break even, or to achieve a target operating income or target net income. Managers also use CVP analysis to guide other decisions, many of them strategic decisions. Consider a decision about choosing additional features for an existing product. Different choices can affect selling prices, variable cost per unit, fixed costs, units sold, and operating income. CVP analysis helps managers make product decisions by estimating the expected profitability of these choices.

Strategic decisions invariably entail risk. CVP analysis can be used to evaluate how operating income will be affected if the original predicted data are not achieved—say, if sales are 10% lower than estimated. Evaluating this risk affects other strategic decisions a company might make. For example, if the probability of a decline in sales seems high, a manager may take actions to change the cost structure to have more variable costs and fewer fixed costs. We return to our GMAT Success example to illustrate how CVP analysis can be used for strategic decisions concerning advertising and selling price.

Decision to Advertise

Suppose Emma anticipates selling 40 units at the fair. Exhibit 3-3 indicates that Emma's operating income will be $1,200. Emma is considering placing an advertisement describing the product and its features in the fair brochure. The advertisement will be a fixed cost of $500. Emma thinks that advertising will increase sales by 10% to 44 packages. Should Emma advertise? The following table presents the CVP analysis.

	40 Packages Sold with No Advertising (1)	44 Packages Sold with Advertising (2)	Difference (3) = (2) − (1)
Revenues ($200 × 40; $200 × 44)	$8,000	$8,800	$ 800
Variable costs ($120 × 40; $120 × 44)	4,800	5,280	480
Contribution margin ($80 × 40; $80 × 44)	3,200	3,520	320
Fixed costs	2,000	2,500	500
Operating income	$1,200	$1,020	$(180)

Operating income will decrease from $1,200 to $1,020, so Emma should not advertise. Note that Emma could focus only on the difference column and come to the same conclusion: If Emma advertises, contribution margin will increase by $320 (revenues, $800 − variable costs, $480), and fixed costs will increase by $500, resulting in a $180 decrease in operating income.

As you become more familiar with CVP analysis, try evaluating decisions based on differences rather than mechanically working through the contribution income statement. Analyzing differences gets to the heart of CVP analysis and sharpens intuition by focusing only on the revenues and costs that will change as a result of a decision.

Decision to Reduce Selling Price

Having decided not to advertise, Emma is contemplating whether to reduce the selling price to $175. At this price, she thinks she will sell 50 units. At this quantity, the test-prep package wholesaler who supplies GMAT Success will sell the packages to Emma for $115 per unit instead of $120. Should Emma reduce the selling price?

Contribution margin from lowering price to $175: ($175 − $115) per unit × 50 units	$3,000
Contribution margin from maintaining price at $200: ($200 − $120) per unit × 40 units	3,200
Change in contribution margin from lowering price	$ (200)

Decreasing the price will reduce contribution margin by $200 and, because the fixed costs of $2,000 will not change, it will also reduce operating income by $200. Emma should not reduce the selling price.

Determining Target Prices

Emma could also ask "At what price can I sell 50 units (purchased at $115 per unit) and continue to earn an operating income of $1,200?" The answer is $179, as the following calculations show.

	Target operating income	$1,200
	Add fixed costs	2,000
	Target contribution margin	$3,200
	Divided by number of units sold	÷50 units
	Target contribution margin per unit	$ 64
	Add variable cost per unit	115
	Target selling price	$ 179
Proof:	Revenues, $179 per unit × 50 units	$8,950
	Variable costs, $115 per unit × 50 units	5,750
	Contribution margin	3,200
	Fixed costs	2,000
	Operating income	$1,200

Emma should also examine the effects of other decisions, such as simultaneously increasing advertising costs and lowering prices. In each case, Emma will compare the changes in contribution margin (through the effects on selling prices, variable costs, and quantities of units sold) to the changes in fixed costs, and she will choose the alternative that provides the highest operating income.

Decision Point

How do managers use CVP analysis to make decisions?

Sensitivity Analysis and Margin of Safety

Learning Objective 5

Explain how sensitivity analysis helps managers cope with uncertainty

. . . determine the effect on operating income of different assumptions

Before choosing strategies and plans about how to implement strategies, managers frequently analyze the sensitivity of their decisions to changes in underlying assumptions. **Sensitivity analysis** is a "what-if" technique that managers use to examine how an outcome will change if the original predicted data are not achieved or if an underlying assumption changes. In the context of CVP analysis, sensitivity analysis answers questions such as, "What will operating income be if the quantity of units sold decreases by 5% from the original prediction?" and "What will operating income be if variable cost per unit increases by 10%?" Sensitivity analysis broadens managers' perspectives to possible outcomes that might occur *before* costs are committed.

Electronic spreadsheets, such as Excel, enable managers to conduct CVP-based sensitivity analyses in a systematic and efficient way. Using spreadsheets, managers can conduct sensitivity analysis to examine the effect and interaction of changes in selling price, variable cost per unit, fixed costs, and target operating income. Exhibit 3-4 displays a spreadsheet for the GMAT Success example.

Using the spreadsheet, Emma can immediately see how many units she needs to sell to achieve particular operating-income levels, given alternative levels of fixed costs and variable cost per unit that she may face. For example, 32 units must be sold to earn an

Exhibit 3-4

Spreadsheet Analysis of CVP Relationships for GMAT Success

Home Insert Page Layout Formulas Data Review View

D5 ▼ fx =($A5+D$3)/(F1-$B5)

	A	B	C	D	E	F
1			Number of units required to be sold at $200			
2			Selling Price to Earn Target Operating Income of			
3		Variable Costs	$0	$1,200	$1,600	$2,000
4	Fixed Costs	per Unit	(Breakeven point)			
5	$2,000	$100	20	32[a]	36	40
6	$2,000	$120	25	40	45	50
7	$2,000	$150	40	64	72	80
8	$2,400	$100	24	36	40	44
9	$2,400	$120	30	45	50	55
10	$2,400	$150	48	72	80	88
11	$2,800	$100	28	40	44	48
12	$2,800	$120	35	50	55	60
13	$2,800	$150	56	80	88	96
14						

[a]Number of units required to be sold $= \dfrac{\text{Fixed costs + Target operating income}}{\text{Contribution margin per unit}} = \dfrac{\$2,000 + \$1,200}{\$200 - \$100} = 32$

operating income of $1,200 if fixed costs are $2,000 and variable cost per unit is $100. Emma can also use Exhibit 3-4 to determine that she needs to sell 56 units to break even if fixed cost of the booth rental at the Chicago fair is raised to $2,800 and if the variable cost per unit charged by the test-prep package supplier increases to $150. Emma can use information about costs and sensitivity analysis, together with realistic predictions about how much she can sell to decide if she should rent a booth at the fair.

Another aspect of sensitivity analysis is **margin of safety**:

$$\text{Margin of safety} = \text{Budgeted (or actual) revenues} - \text{Breakeven revenues}$$
$$\text{Margin of safety (in units)} = \text{Budgeted (or actual) sales quantity} - \text{Breakeven quantity}$$

The margin of safety answers the "what-if" question: If budgeted revenues are above breakeven and drop, how far can they fall below budget before the breakeven point is reached? Sales might decrease as a result of a competitor introducing a better product, or poorly executed marketing programs, and so on. Assume that Emma has fixed costs of $2,000, a selling price of $200, and variable cost per unit of $120. From Exhibit 3-1, if Emma sells 40 units, budgeted revenues are $8,000 and budgeted operating income is $1,200. The breakeven point is 25 units or $5,000 in total revenues.

$$\text{Margin of safety} = \begin{matrix}\text{Budgeted}\\\text{revenues}\end{matrix} - \begin{matrix}\text{Breakeven}\\\text{revenues}\end{matrix} = \$8,000 - \$5,000 = \$3,000$$

$$\begin{matrix}\text{Margin of}\\\text{safety (in units)}\end{matrix} = \begin{matrix}\text{Budgeted}\\\text{sales (units)}\end{matrix} - \begin{matrix}\text{Breakeven}\\\text{sales (units)}\end{matrix} = 40 - 25 = 15 \text{ units}$$

Sometimes margin of safety is expressed as a percentage:

$$\text{Margin of safety percentage} = \frac{\text{Margin of safety in dollars}}{\text{Budgeted (or actual) revenues}}$$

In our example, margin of safety percentage $= \dfrac{\$3,000}{\$8,000} = 37.5\%$

This result means that revenues would have to decrease substantially, by 37.5%, to reach breakeven revenues. The high margin of safety gives Emma confidence that she is unlikely to suffer a loss.

If, however, Emma expects to sell only 30 units, budgeted revenues would be \$6,000 (\$200 per unit $\times$ 30 units) and the margin of safety would equal:

$$\text{Budgeted revenues} - \text{Breakeven revenues} = \$6{,}000 - \$5{,}000 = \$1{,}000$$

$$\begin{array}{c}\text{Margin of}\\\text{safety percentage}\end{array} = \frac{\text{Margin of safety in dollars}}{\text{Budgeted (or actual) revenues}} = \frac{\$1{,}000}{\$6{,}000} = 16.67\%$$

The analysis implies that if revenues decrease by more than 16.67%, Emma would suffer a loss. A low margin of safety increases the risk of a loss. If Emma does not have the tolerance for this level of risk, she will prefer not to rent a booth at the fair.

Sensitivity analysis is a simple approach to recognizing **uncertainty**, which is the possibility that an actual amount will deviate from an expected amount. Sensitivity analysis gives managers a good feel for the risks involved. A more comprehensive approach to recognizing uncertainty is to compute expected values using probability distributions. This approach is illustrated in the appendix to this chapter.

Decision Point

What can managers do to cope with uncertainty or changes in underlying assumptions?

Cost Planning and CVP

Managers have the ability to choose the levels of fixed and variable costs in their cost structures. This is a strategic decision. In this section, we describe various factors that managers and management accountants consider as they make this decision.

Learning Objective 6

Use CVP analysis to plan variable and fixed costs

. . . compare risk of losses versus higher returns

Alternative Fixed-Cost/Variable-Cost Structures

CVP-based sensitivity analysis highlights the risks and returns as fixed costs are substituted for variable costs in a company's cost structure. In Exhibit 3-4, compare line 6 and line 11.

			Number of units required to be sold at \$200 selling price to earn target operating income of	
	Fixed Cost	Variable Cost	\$0 (Breakeven point)	\$2,000
Line 6	\$2,000	\$120	25	50
Line 11	\$2,800	\$100	28	48

Compared to line 6, line 11, with higher fixed costs, has more risk of loss (has a higher breakeven point) but requires fewer units to be sold (48 versus 50) to earn operating income of \$2,000. CVP analysis can help managers evaluate various fixed-cost/variable-cost structures. We next consider the effects of these choices in more detail. Suppose the Chicago college fair organizers offer Emma three rental alternatives:

Option 1: \$2,000 fixed fee

Option 2: \$800 fixed fee plus 15% of GMAT Success revenues

Option 3: 25% of GMAT Success revenues with no fixed fee

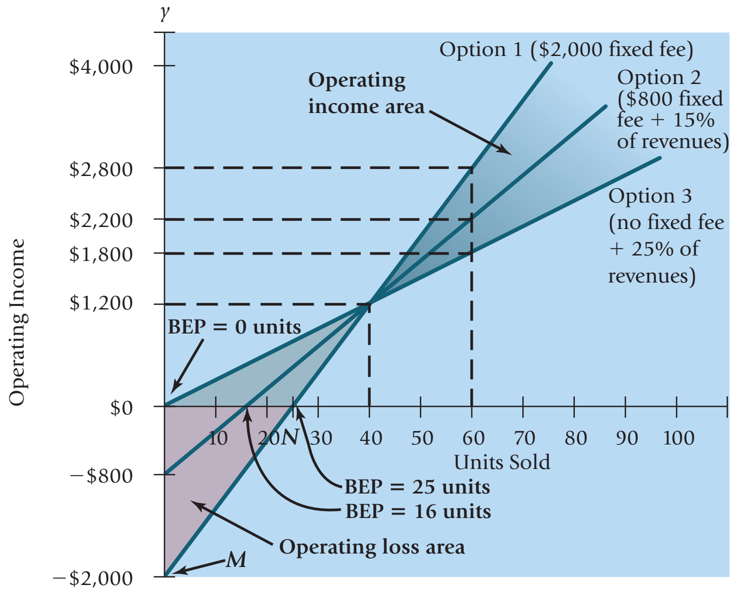
Emma's variable cost per unit is \$120. Emma is interested in how her choice of a rental agreement will affect the income she earns and the risks she faces. Exhibit 3-5 graphically depicts the profit-volume relationship for each option. The line representing the relationship between units sold and operating income for Option 1 is the same as the line in the PV graph shown in Exhibit 3-3 (fixed costs of \$2,000 and contribution margin per unit of \$80). The line representing Option 2 shows fixed costs of \$800 and a contribution margin per unit of \$50 [selling price, \$200, minus variable cost per unit, \$120, minus variable rental fees per unit, \$30, (0.15 $\times$ \$200)]. The line representing Option 3 has fixed costs of \$0 and a contribution margin per unit of \$30 [\$200 $-$ \$120 $-$ \$50 (0.25 $\times$ \$200)].

Option 3 has the lowest breakeven point (0 units), and Option 1 has the highest breakeven point (25 units). Option 1 has the highest risk of loss if sales are low, but it also has the highest contribution margin per unit (\$80) and hence the highest operating income when sales are high (greater than 40 units).

The choice among Options 1, 2, and 3 is a strategic decision that Emma faces. As in most strategic decisions, what she decides now will significantly affect her operating

Exhibit 3-5

Profit-Volume Graph for Alternative Rental Options for GMAT Success



BEP = Breakeven point

income (or loss), depending on the demand for GMAT Success. Faced with this uncertainty, Emma's choice will be influenced by her confidence in the level of demand for GMAT Success and her willingness to risk losses if demand is low. For example, if Emma's tolerance for risk is high, she will choose Option 1 with its high potential rewards. If, however, Emma is averse to taking risk, she will prefer Option 3, where the rewards are smaller if sales are high but where she never suffers a loss if sales are low.

Operating Leverage

The risk-return trade-off across alternative cost structures can be measured as *operating leverage*. **Operating leverage** describes the effects that fixed costs have on changes in operating income as changes occur in units sold and contribution margin. Organizations with a high proportion of fixed costs in their cost structures, as is the case under Option 1, have high operating leverage. The line representing Option 1 in Exhibit 3-5 is the steepest of the three lines. Small increases in sales lead to large increases in operating income. Small decreases in sales result in relatively large decreases in operating income, leading to a greater risk of operating losses. *At any given level of sales*,

$$\text{Degree of operating leverage} = \frac{\text{Contribution margin}}{\text{Operating income}}$$

The following table shows the **degree of operating leverage** at sales of 40 units for the three rental options.

	Option 1	Option 2	Option 3
1. Contribution margin per unit (p. 75)	$ 80	$ 50	$ 30
2. Contribution margin (row 1 × 40 units)	$3,200	$2,000	$1,200
3. Operating income (from Exhibit 3-5)	$1,200	$1,200	$1,200
4. Degree of operating leverage (row 2 ÷ row 3)	$\frac{\$3,200}{\$1,200} = 2.67$	$\frac{\$2,000}{\$1,200} = 1.67$	$\frac{\$1,200}{\$1,200} = 1.00$

These results indicate that, when sales are 40 units, a percentage change in sales and contribution margin will result in 2.67 times that percentage change in operating income for Option 1, but the same percentage change (1.00) in operating income for Option 3. Consider, for example, a sales increase of 50% from 40 to 60 units. Contribution margin will increase by 50% under each option. Operating income, however, will increase by 2.67 × 50% = 133% from $1,200 to $2,800 in Option 1, but it will increase by only 1.00 × 50% = 50% from $1,200 to $1,800 in Option 3 (see Exhibit 3-5). The degree of operating leverage at a given level of sales helps managers calculate the effect of sales fluctuations on operating income.

Keep in mind that, in the presence of fixed costs, the degree of operating leverage is different at different levels of sales. For example, at sales of 60 units, the degree of operating leverage under each of the three options is as follows:

	Option 1	Option 2	Option 3
1. Contribution margin per unit (p. 75)	$ 80	$ 50	$ 30
2. Contribution margin (row 1 × 60 units)	$4,800	$3,000	$1,800
3. Operating income (from Exhibit 3-5)	$2,800	$2,200	$1,800
4. Degree of operating leverage (row 2 ÷ row 3)	$\frac{\$4,800}{\$2,800} = 1.71$	$\frac{\$3,000}{\$2,200} = 1.36$	$\frac{\$1,800}{\$1,800} = 1.00$

The degree of operating leverage decreases from 2.67 (at sales of 40 units) to 1.71 (at sales of 60 units) under Option 1 and from 1.67 to 1.36 under Option 2. In general, whenever there are fixed costs, the degree of operating leverage decreases as the level of sales increases beyond the breakeven point. If fixed costs are $0 as in Option 3, contribution margin equals operating income, and the degree of operating leverage equals 1.00 at all sales levels.

But why must managers monitor operating leverage carefully? Again, consider companies such as General Motors, Global Crossing, US Airways, United Airlines, and WorldCom. Their high operating leverage was a major reason for their financial problems. Anticipating high demand for their services, these companies borrowed money to acquire assets, resulting in high fixed costs. As sales declined, these companies suffered losses and could not generate sufficient cash to service their interest and debt, causing them to seek bankruptcy protection. Managers and management accountants should always evaluate how the level of fixed costs and variable costs they choose will affect the risk-return trade-off. See Concepts in Action, page 78, for another example of the risks of high fixed costs.

What actions are managers taking to reduce their fixed costs? Many companies are moving their manufacturing facilities from the United States to lower-cost countries, such as Mexico and China. To substitute high fixed costs with lower variable costs, companies are purchasing products from lower-cost suppliers instead of manufacturing products themselves. These actions reduce both costs and operating leverage. More recently, General Electric and Hewlett-Packard began outsourcing service functions, such as post-sales customer service, by shifting their customer call centers to countries, such as India, where costs are lower. These decisions by companies are not without controversy. Some economists argue that outsourcing helps to keep costs, and therefore prices, low and enables U.S. companies to remain globally competitive. Others argue that outsourcing reduces job opportunities in the United States and hurts working-class families.

Decision Point

How should managers choose among different variable-cost/fixed-cost structures?

Effects of Sales Mix on Income

Learning Objective 7

Apply CVP analysis to a company producing multiple products

. . . assume sales mix of products remains constant as total units sold changes

Sales mix is the quantities (or proportion) of various products (or services) that constitute total unit sales of a company. Suppose Emma is now budgeting for a subsequent college fair in New York. She plans to sell two different test-prep packages—GMAT Success and GRE Guarantee—and budgets the following:

	GMAT Success	GRE Guarantee	Total
Expected sales	60	40	100
Revenues, $200 and $100 per unit	$12,000	$4,000	$16,000
Variable costs, $120 and $70 per unit	7,200	2,800	10,000
Contribution margin, $80 and $30 per unit	$ 4,800	$1,200	6,000
Fixed costs			4,500
Operating income			$ 1,500

Concepts in Action

Fixed Costs, Variable Costs, and the Future of Radio

Building up too much fixed costs can be hazardous to a company's health. Because fixed costs, unlike variable costs, do not automatically decrease as volume declines, companies with too much fixed costs can lose a considerable amount of money during lean times. Sirius XM, the satellite radio broadcaster, learned this lesson the hard way.

To begin broadcasting in 2001, both Sirius Satellite Radio and XM Satellite Radio—the two companies now comprising Sirius XM—spent billions of dollars on broadcasting licenses, space satellites, and other technology infrastructure. Once operational, the companies also spent billions on other fixed items such as programming and content (including Howard Stern and Major League Baseball), satellite transmission, and R&D. In contrast, variable costs were minimal, consisting mainly of artist-royalty fees and customer service and billing. In effect, this created a business model with a high operating leverage—that is, the companies' cost structure had a very significant proportion of fixed costs. As such, profitability could only be achieved by amassing millions of paid subscribers and selling advertising.

The competitive disadvantage of this highly-leveraged business model was nearly disastrous. Despite amassing more than 14 million subscribers, over the years Sirius and XM rang up $3 billion in debt and tallied cumulative operating losses in excess of $10 billion. Operating leverage, and the threat of bankruptcy, forced the merger of Sirius and XM in 2007, and since then the combined entity has struggled to cut costs, refinance its sizable debt, and reap the profits from over 18 million monthly subscribers.

While satellite radio has struggled under the weight of too much fixed cost, Internet radio had the opposite problem—too much variable costs. But "How?" you ask. Don't variable costs only increase as revenues increase? Yes, but if the revenue earned is less than the variable cost, an increase in revenue can lead to bankruptcy. This is almost what happened to Pandora, the Internet radio service.

Pandora launched in 2005 with only $9.3 million in venture capital. Available free over the Internet, Pandora earned revenue in three ways: advertising on its Web site, subscription fees from users who wanted to opt-out of advertising, and affiliate fees from iTunes and Amazon.com. Pandora had low fixed costs but high variable costs for streaming and performance royalties. Over time, as Pandora's popular service attracted millions of loyal listeners, its costs for performance royalties—set by the Copyright Royalty Board on a per song basis—far exceeded its revenues from advertising and subscriptions. As a result, even though royalty rates were only a fraction of a cent, Pandora lost more and more money each time it played another song!

In 2009, Pandora avoided bankruptcy by renegotiating a lower per-song royalty rate in exchange for at least 25% of its U.S. revenue annually. Further, Pandora began charging its most frequent users a small fee and also started increasing its advertising revenue.

Sources: Birger, Jon. 2009. Mel Karmazian fights to rescue Sirius. *Fortune*, March 16; Clifford, Stephanie. 2007. Pandora's long strange trip. *Inc.*, October 1; Pandora: Royalties kill the web radio star? (A). Harvard Business School Case No. 9-310-026; Satellite radio: An industry case study. Kellogg School of Management, Northwestern University. Case No. 5-206-255; XM satellite radio (A). Harvard Business School Case No. 9-504-009.

What is the breakeven point? In contrast to the single-product (or service) situation, the total number of units that must be sold to break even in a multiproduct company depends on the sales mix—the combination of the number of units of GMAT Success sold and the number of units of GRE Guarantee sold. We assume that the budgeted sales mix (60 units of GMAT Success sold for every 40 units of GRE Guarantee sold, that is, a ratio of 3:2) will not change at different levels of total unit sales. That is, we think of Emma selling a bundle of 3 units of GMAT Success and 2 units of GRE Guarantee. (Note that this does not mean that Emma physically bundles the two products together into one big package.)

Each bundle yields a contribution margin of $300 calculated as follows:

	Number of Units of GMAT Success and GRE Guarantee in Each Bundle	Contribution Margin per Unit for GMAT Success and GRE Guarantee	Contribution Margin of the Bundle
GMAT Success	3	$80	$240
GRE Guarantee	2	30	60
Total			$300

To compute the breakeven point, we calculate the number of bundles Emma needs to sell.

$$\text{Breakeven point in bundles} = \frac{\text{Fixed costs}}{\text{Contribution margin per bundle}} = \frac{\$4{,}500}{\$300 \text{ per bundle}} = 15 \text{ bundles}$$

Breakeven point in units of GMAT Success and GRE Guarantee is as follows:

GMAT Success: 15 bundles × 3 units of GMAT Success per bundle	45 units
GRE Guarantee: 15 bundles × 2 units of GRE Guarantee per bundle	30 units
Total number of units to break even	75 units

Breakeven point in dollars for GMAT Success and GRE Guarantee is as follows:

GMAT Success: 45 units × $200 per unit	$ 9,000
GRE Guarantee: 30 units × $100 per unit	3,000
Breakeven revenues	$12,000

When there are multiple products, it is often convenient to use contribution margin percentage. Under this approach, Emma first calculates the revenues from selling a bundle of 3 units of GMAT Success and 2 units of GRE Guarantee:

	Number of Units of GMAT Success and GRE Guarantee in Each Bundle	Selling Price for GMAT Success and GRE Guarantee	Revenue of the Bundle
GMAT Success	3	$200	$600
GRE Guarantee	2	100	200
Total			$800

$$\text{Contribution margin percentage for the bundle} = \frac{\text{Contribution margin of the bundle}}{\text{Revenue of the bundle}} = \frac{\$300}{\$800} = 0.375 \text{ or } 37.5\%$$

$$\text{Breakeven revenues} = \frac{\text{Fixed costs}}{\text{Contribution margin \% for the bundle}} = \frac{\$4{,}500}{0.375} = \$12{,}000$$

$$\text{Number of bundles required to be sold to break even} = \frac{\text{Breakeven revenues}}{\text{Revenue per bundle}} = \frac{\$12{,}000}{\$800 \text{ per bundle}} = 15 \text{ bundles}$$

The breakeven point in units and dollars for GMAT Success and GRE Guarantee are as follows:

GMAT Success: 15 bundles × 3 units of GMAT Success per bundle = 45 units × $200 per unit = $9,000

GRE Guarantee: 15 bundles × 2 units of GRE Guarantee per bundle = 30 units × $100 per unit = $3,000

Recall that in all our calculations we have assumed that the budgeted sales mix (3 units of GMAT Success for every 2 units of GRE Guarantee) will not change at different levels of total unit sales.

Of course, there are many different sales mixes (in units) that result in a contribution margin of $4,500 and cause Emma to break even, as the following table shows:

Sales Mix (Units)		Contribution Margin from		
GMAT Success (1)	GRE Guarantee (2)	GMAT Success (3) = $80 × (1)	GRE Guarantee (4) = $30 × (2)	Total Contribution Margin (5) = (3) + (4)
48	22	$3,840	$ 660	$4,500
36	54	2,880	1,620	4,500
30	70	2,400	2,100	4,500

If for example, the sales mix changes to 3 units of GMAT Success for every 7 units of GRE Guarantee, the breakeven point increases from 75 units to 100 units, comprising 30 units of GMAT Success and 70 units of GRE Guarantee. The breakeven quantity increases because the sales mix has shifted toward the lower-contribution-margin product, GRE Guarantee ($30 per unit compared to GMAT Success's $80 per unit). In general, for any given total quantity of units sold, as the sales mix shifts toward units with lower contribution margins (more units of GRE Guarantee compared to GMAT Success), operating income will be lower.

How do companies choose their sales mix? They adjust their mix to respond to demand changes. For example, as gasoline prices increase and customers want smaller cars, auto companies shift their production mix to produce smaller cars.

Decision Point

How can CVP analysis be applied to a company producing multiple products?

The multi-product case has two cost drivers, GMAT Success and GRE Guarantee. It shows how CVP and breakeven analysis can be adapted to the case of multiple cost drivers. The key point is that many different combinations of cost drivers can result in a given contribution margin.

CVP Analysis in Service and Nonprofit Organizations

Thus far, our CVP analysis has focused on a merchandising company. CVP can also be applied to decisions by manufacturing companies like BMW, service companies like Bank of America, and nonprofit organizations like the United Way. To apply CVP analysis in service and nonprofit organizations, we need to focus on measuring their output, which is different from the tangible units sold by manufacturing and merchandising companies. Examples of output measures in various service and nonprofit industries are as follows:

Industry	Measure of Output
Airlines	Passenger miles
Hotels/motels	Room-nights occupied
Hospitals	Patient days
Universities	Student credit-hours

Consider an agency of the Massachusetts Department of Social Welfare with a $900,000 budget appropriation (its revenues) for 2011. This nonprofit agency's purpose is to assist handicapped people seeking employment. On average, the agency supplements each person's income by $5,000 annually. The agency's only other costs are fixed costs of rent and administrative salaries equal to $270,000. The agency manager wants to know how many people could be assisted in 2011. We can use CVP analysis here by setting operating income to $0. Let Q be the number of handicapped people to be assisted:

$$\text{Revenues} - \text{Variable costs} - \text{Fixed costs} = 0$$
$$\$900{,}000 - \$5{,}000\,Q - \$270{,}000 = 0$$
$$\$5{,}000\,Q = \$900{,}000 - \$270{,}000 = \$630{,}000$$
$$Q = \$630{,}000 \div \$5{,}000 \text{ per person} = 126 \text{ people}$$

Suppose the manager is concerned that the total budget appropriation for 2012 will be reduced by 15% to $900,000 × (1 − 0.15) = $765,000. The manager wants to know how many handicapped people could be assisted with this reduced budget. Assume the same amount of monetary assistance per person:

$$\$765{,}000 - \$5{,}000\,Q - \$270{,}000 = 0$$
$$\$5{,}000Q = \$765{,}000 - \$270{,}000 = \$495{,}000$$
$$Q = \$495{,}000 \div \$5{,}000 \text{ per person} = 99 \text{ people}$$

Note the following two characteristics of the CVP relationships in this nonprofit situation:

1. The percentage drop in the number of people assisted, (126 − 99) ÷ 126, or 21.4%, is greater than the 15% reduction in the budget appropriation. It is greater because the $270,000 in fixed costs still must be paid, leaving a proportionately lower budget to assist people. The percentage drop in service exceeds the percentage drop in budget appropriation.
2. Given the reduced budget appropriation (revenues) of $765,000, the manager can adjust operations to stay within this appropriation in one or more of three basic ways: (a) reduce the number of people assisted from the current 126, (b) reduce the variable cost (the extent of assistance per person) from the current $5,000 per person, or (c) reduce the total fixed costs from the current $270,000.

Contribution Margin Versus Gross Margin

In the following equations, we clearly distinguish contribution margin, which provides information for CVP analysis, from gross margin, a measure of competitiveness, as defined in Chapter 2.

$$\text{Gross margin} = \text{Revenues} - \text{Cost of goods sold}$$
$$\text{Contribution margin} = \text{Revenues} - \text{All variable costs}$$

Gross margin measures how much a company can charge for its products over and above the cost of acquiring or producing them. Companies, such as branded pharmaceuticals, have high gross margins because their products provide unique and distinctive benefits to consumers. Products such as televisions that operate in competitive markets have low gross margins. Contribution margin indicates how much of a company's revenues are available to cover fixed costs. It helps in assessing risk of loss. Risk of loss is low (high) if, when sales are low, contribution margin exceeds (is less than) fixed costs. Gross margin and contribution margin are related but give different insights. For example, a company operating in a competitive market with a low gross margin will have a low risk of loss if its fixed costs are small.

Consider the distinction between gross margin and contribution margin in the context of manufacturing companies. In the manufacturing sector, contribution margin and gross margin differ in two respects: fixed manufacturing costs and variable nonmanufacturing costs. The following example (figures assumed) illustrates this difference:

Contribution Income Statement Emphasizing Contribution Margin (in 000s)			**Financial Accounting Income Statement Emphasizing Gross Margin (in 000s)**	
Revenues		$1,000	Revenues	$1,000
Variable manufacturing costs	$250		Cost of goods sold (variable manufacturing costs, $250 + fixed manufacturing costs, $160)	410
Variable nonmanufacturing costs	270	520		
Contribution margin		480	Gross margin	590
Fixed manufacturing costs	160		Nonmanufacturing costs	
Fixed nonmanufacturing costs	138	298	(variable, $270 + fixed $138)	408
Operating income		$ 182	Operating income	$ 182

Fixed manufacturing costs of $160,000 are not deducted from revenues when computing contribution margin but are deducted when computing gross margin. Cost of goods sold in a manufacturing company includes all variable manufacturing costs and all fixed manufacturing costs ($250,000 + $160,000). Variable nonmanufacturing costs (such as commissions paid to salespersons) of $270,000 are deducted from revenues when computing contribution margin but are not deducted when computing gross margin.

Like contribution margin, gross margin can be expressed as a total, as an amount per unit, or as a percentage. For example, the **gross margin percentage** is the gross margin divided by revenues—59% ($590 ÷ $1,000) in our manufacturing-sector example.

One reason why gross margin and contribution margin are confused with each other is that the two are identical in the case of merchandising companies. That's because cost of goods sold equals the variable cost of goods purchased (and subsequently sold).

Problem for Self-Study

Wembley Travel Agency specializes in flights between Los Angeles and London. It books passengers on United Airlines at $900 per round-trip ticket. Until last month, United paid Wembley a commission of 10% of the ticket price paid by each passenger. This commission was Wembley's only source of revenues. Wembley's fixed costs are $14,000 per month (for salaries, rent, and so on), and its variable costs are $20 per ticket purchased for a passenger. This $20 includes a $15 per ticket delivery fee paid to Federal Express. (To keep the analysis simple, we assume each round-trip ticket purchased is delivered in a separate package. Thus, the $15 delivery fee applies to each ticket.)

United Airlines has just announced a revised payment schedule for all travel agents. It will now pay travel agents a 10% commission per ticket up to a maximum of $50. Any ticket costing more than $500 generates only a $50 commission, regardless of the ticket price.

Required

1. Under the old 10% commission structure, how many round-trip tickets must Wembley sell each month (a) to break even and (b) to earn an operating income of $7,000?
2. How does United's revised payment schedule affect your answers to (a) and (b) in requirement 1?

Solution

1. Wembley receives a 10% commission on each ticket: 10% × $900 = $90. Thus,

$$\begin{aligned}\text{Selling price} &= \$90 \text{ per ticket}\\ \text{Variable cost per unit} &= \$20 \text{ per ticket}\\ \text{Contribution margin per unit} &= \$90 - \$20 = \$70 \text{ per ticket}\\ \text{Fixed costs} &= \$14{,}000 \text{ per month}\end{aligned}$$

a. $$\text{Breakeven number of tickets} = \frac{\text{Fixed costs}}{\text{Contribution margin per unit}} = \frac{\$14{,}000}{\$70 \text{ per ticket}} = 200 \text{ tickets}$$

b. When target operating income = $7,000 per month,

$$\begin{aligned}\text{Quantity of tickets required to be sold} &= \frac{\text{Fixed costs} + \text{Target operating income}}{\text{Contribution margin per unit}}\\ &= \frac{\$14{,}000 + \$7{,}000}{\$70 \text{ per ticket}} = \frac{\$21{,}000}{\$70 \text{ per ticket}} = 300 \text{ tickets}\end{aligned}$$

2. Under the new system, Wembley would receive only $50 on the $900 ticket. Thus,

$$\begin{aligned}\text{Selling price} &= \$50 \text{ per ticket}\\ \text{Variable cost per unit} &= \$20 \text{ per ticket}\\ \text{Contribution margin per unit} &= \$50 - \$20 = \$30 \text{ per ticket}\\ \text{Fixed costs} &= \$14{,}000 \text{ per month}\end{aligned}$$

a. $$\text{Breakeven number of tickets} = \frac{\$14{,}000}{\$30 \text{ per ticket}} = 467 \text{ tickets (rounded up)}$$

b. $$\text{Quantity of tickets required to be sold} = \frac{\$21{,}000}{\$30 \text{ per ticket}} = 700 \text{ tickets}$$

The $50 cap on the commission paid per ticket causes the breakeven point to more than double (from 200 to 467 tickets) and the tickets required to be sold to earn $7,000 per month to also more than double (from 300 to 700 tickets). As would be expected, travel agents reacted very negatively to the United Airlines announcement to change commission payments. Unfortunately for travel agents, other airlines also changed their commission structure in similar ways.

Decision Points

The following question-and-answer format summarizes the chapter's learning objectives. Each decision presents a key question related to a learning objective. The guidelines are the answer to that question.

Decision	Guidelines
1. How can CVP analysis assist managers?	CVP analysis assists managers in understanding the behavior of a product's or service's total costs, total revenues, and operating income as changes occur in the output level, selling price, variable costs, or fixed costs.
2. How can managers determine the breakeven point or the output needed to achieve a target operating income?	The breakeven point is the quantity of output at which total revenues equal total costs. The three methods for computing the breakeven point and the quantity of output to achieve target operating income are the equation method, the contribution margin method, and the graph method. Each method is merely a restatement of the others. Managers often select the method they find easiest to use in the specific decision situation.
3. How can managers incorporate income taxes into CVP analysis?	Income taxes can be incorporated into CVP analysis by using target net income to calculate the corresponding target operating income. The breakeven point is unaffected by income taxes because no income taxes are paid when operating income equals zero.
4. How do managers use CVP analysis to make decisions?	Managers compare how revenues, costs, and contribution margins change across various alternatives. They then choose the alternative that maximizes operating income.
5. What can managers do to cope with uncertainty or changes in underlying assumptions?	Sensitivity analysis, a "what-if" technique, examines how an outcome will change if the original predicted data are not achieved or if an underlying assumption changes. When making decisions, managers use CVP analysis to compare contribution margins and fixed costs under different assumptions. Managers also calculate the margin of safety equal to budgeted revenues minus breakeven revenues.
6. How should managers choose between different variable-cost/fixed-cost structures?	Choosing the variable-cost/fixed-cost structure is a strategic decision for companies. CVP analysis highlights the risk of losses when revenues are low and the upside profits when revenues are high for different proportions of variable and fixed costs in a company's cost structure.
7. How can CVP analysis be applied to a company producing multiple products?	CVP analysis can be applied to a company producing multiple products by assuming the sales mix of products sold remains constant as the total quantity of units sold changes.

Appendix

Decision Models and Uncertainty

This appendix explores the characteristics of uncertainty, describes an approach managers can use to make decisions in a world of uncertainty, and illustrates the insights gained when uncertainty is recognized in CVP analysis.

Coping with Uncertainty[2]

In the face of uncertainty, managers rely on decision models to help them make the right choices.

Role of a Decision Model

Uncertainty is the possibility that an actual amount will deviate from an expected amount. In the GMAT Success example, Emma might forecast sales at 42 units, but actual sales might turn out to be 30 units or 60 units. A decision model helps managers deal with such uncertainty. It is a formal method for making a choice, commonly involving both quantitative and qualitative analyses. The quantitative analysis usually includes the following steps:

Step 1: Identify a choice criterion. A **choice criterion** is an objective that can be quantified such as maximize income or minimize costs. Managers use the choice criterion to choose the best alternative action. Emma's choice criterion is to maximize expected operating income at the Chicago college fair.

Step 2: Identify the set of alternative actions that can be taken. We use the letter *a* with subscripts $_1$, $_2$, and $_3$ to distinguish each of Emma's three possible actions:

a_1 = Pay \$2,000 fixed fee
a_2 = Pay \$800 fixed fee plus 15% of GMAT Success revenues
a_3 = Pay 25% of GMAT Success revenues with no fixed fee

Step 3: Identify the set of events that can occur. An **event** is a possible relevant occurrence, such as the actual number of GMAT Success packages Emma might sell at the fair. The set of events should be mutually exclusive and collectively exhaustive. Events are mutually exclusive if they cannot occur at the same time. Events are collectively exhaustive if, taken together, they make up the entire set of possible relevant occurrences (no other event can occur). Examples of mutually exclusive and collectively exhaustive events are growth, decline, or no change in industry demand, and increase, decrease, or no change in interest rates. Only one event out of the entire set of mutually exclusive and collectively exhaustive events will actually occur.

Suppose Emma's only uncertainty is the number of units of GMAT Success that she can sell. For simplicity, suppose Emma estimates that sales will be either 30 or 60 units. This set of events is mutually exclusive because clearly sales of 30 units and 60 units cannot both occur at the same time. It is collectively exhaustive because under our assumptions, sales cannot be anything other than 30 or 60 units. We use the letter *x* with subscripts $_1$ and $_2$ to distinguish the set of mutually exclusive and collectively exhaustive events:

$$x_1 = 30 \text{ units}$$
$$x_2 = 60 \text{ units}$$

Step 4: Assign a probability to each event that can occur. A **probability** is the likelihood or chance that an event will occur. The decision model approach to coping with uncertainty assigns probabilities to events. A **probability distribution** describes the likelihood, or the probability, that each of the mutually exclusive and collectively exhaustive set of events will occur. In some cases, there will be much evidence to guide the assignment of probabilities. For example, the probability of obtaining heads in the toss of a coin is 1/2 and that of drawing a particular playing card from a standard, well-shuffled deck is 1/52. In business, the probability of having a specified percentage of defective units may be assigned with great confidence on the basis of production experience with thousands of units. In other cases, there will be little evidence supporting estimated probabilities—for example, expected sales of a new pharmaceutical product next year.

Suppose that Emma, on the basis of past experience, assesses a 60% chance, or a 6/10 probability, that she will sell 30 units and a 40% chance, or a 4/10 probability, that she will sell 60 units. Using P*(x)* as the notation for the probability of an event, the probabilities are as follows:

$$P(x_1) = 6/10 = 0.60$$
$$P(x_2) = 4/10 = 0.40$$

The sum of these probabilities must equal 1.00 because these events are mutually exclusive and collectively exhaustive.

[2] The presentation here draws (in part) from teaching notes prepared by R. Williamson.

Exhibit 3-6 Decision Table for GMAT Success

	A	B	C	D	E	F	G	H	I
1	Selling price =	$200			Operating Income				
2	Package cost =	$120			Under Each Possible Event				
3			Percentage						
4		Fixed	of Fair	Event x_1: Units Sold =		30	Event x_2: Units Sold =		60
5	Actions	Fee	Revenues	Probability(x_1) =		0.60	Probability(x_2) =		0.40
6	a_1: Pay $2,000 fixed fee	$2,000	0%	$400[l]			$2,800[m]		
7	a_2: Pay $800 fixed fee plus 15% of revenues	$ 800	15%	$700[n]			$2,200[p]		
8	a_3: Pay 25% of revenues with no fixed fee	$ 0	25%	$900[q]			$1,800[r]		
9									
10	[l]Operating income = ($200 – $120)(30) – $2,000	=	$ 400						
11	[m]Operating income = ($200 – $120)(60) – $2,000	=	$2,800						
12	[n]Operating income = ($200 – $120 – 15% × $200)(30) – $800	=	$ 700						
13	[p]Operating income = ($200 – $120 – 15% × $200)(60) – $800	=	$2,200						
14	[q]Operating income = ($200 – $120 – 25% × $200)(30)	=	$ 900						
15	[r]Operating income = ($200 – $120 – 25% × $200)(60)	=	$1,800						

Step 5: Identify the set of possible outcomes. Outcomes specify, in terms of the choice criterion, the predicted economic results of the various possible combinations of actions and events. In the GMAT Success example, the outcomes are the six possible operating incomes displayed in the decision table in Exhibit 3-6. A **decision table** is a summary of the alternative actions, events, outcomes, and probabilities of events.

Distinguish among actions, events, and outcomes. Actions are decision choices available to managers—for example, the particular rental alternatives that Emma can choose. Events are the set of all relevant occurrences that can happen—for example, the different quantities of GMAT Success packages that may be sold at the fair. The outcome is operating income, which depends both on the action the manager selects (rental alternative chosen) and the event that occurs (the quantity of packages sold).

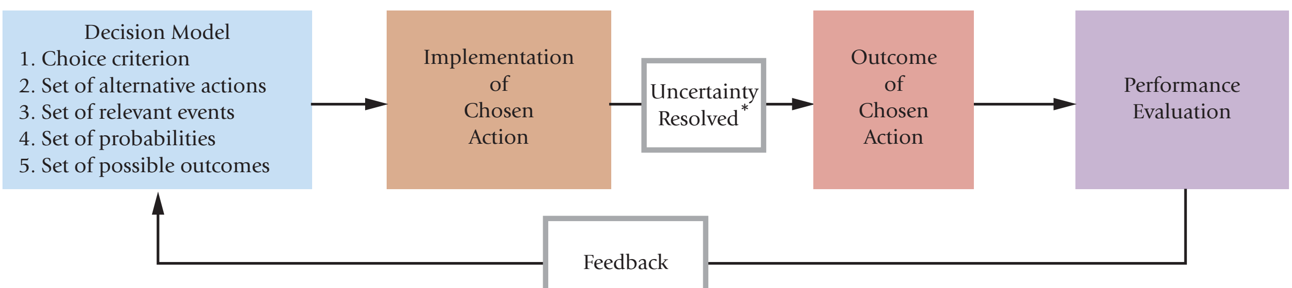
Exhibit 3-7 presents an overview of relationships among a decision model, the implementation of a chosen action, its outcome, and a subsequent performance evaluation. Thoughtful managers step back and evaluate what happened and learn from their experiences. This learning serves as feedback for adapting the decision model for future actions.

Expected Value

An **expected value** is the weighted average of the outcomes, with the probability of each outcome serving as the weight. When the outcomes are measured in monetary terms, expected value is often called **expected monetary value.** Using information in Exhibit 3-6, the expected monetary value of each booth-rental alternative denoted by $E(a_1)$, $E(a_2)$, and $E(a_3)$ is as follows:

Pay $2,000 fixed fee:	$E(a_1) = (0.60 \times \$400) + (0.40 \times \$2{,}800) = \$1{,}360$
Pay $800 fixed fee plus 15% of revenues:	$E(a_2) = (0.60 \times \$700) + (0.40 \times \$2{,}200) = \$1{,}300$
Pay 25% of revenues with no fixed fee:	$E(a_3) = (0.60 \times \$900) + (0.40 \times \$1{,}800) = \$1{,}260$

Exhibit 3-7 A Decision Model and Its Link to Performance Evaluation



*Uncertainty resolved means the event becomes known.

To maximize expected operating income, Emma should select action a_1—pay the fair organizers a $2,000 fixed fee.

To interpret the expected value of selecting action a_1, imagine that Emma attends many fairs, each with the probability distribution of operating incomes given in Exhibit 3-6. For a specific fair, Emma will earn operating income of either $400, if she sells 30 units, or $2,800, if she sells 60 units. But if Emma attends 100 fairs, she will expect to earn $400 operating income 60% of the time (at 60 fairs), and $2,800 operating income 40% of the time (at 40 fairs), for a total operating income of $136,000 ($400 × 60 + $2,800 × 40). The expected value of $1,360 is the operating income per fair that Emma will earn when averaged across all fairs ($136,000 ÷ 100). Of course, in many real-world situations, managers must make one-time decisions under uncertainty. Even in these cases, expected value is a useful tool for choosing among alternatives.

Consider the effect of uncertainty on the preferred action choice. If Emma were certain she would sell only 30 units (that is, $P(x_1) = 1$), she would prefer alternative a_3—pay 25% of revenues with no fixed fee. To follow this reasoning, examine Exhibit 3-6. When 30 units are sold, alternative a_3 yields the maximum operating income of $900. Because fixed costs are $0, booth-rental costs are lower, equal to $1,500 (25% of revenues = 0.25 × $200 per unit × 30 units), when sales are low.

However, if Emma were certain she would sell 60 packages (that is, $P(x_2) = 1$), she would prefer alternative a_1—pay a $2,000 fixed fee. Exhibit 3-6 indicates that when 60 units are sold, alternative a_1 yields the maximum operating income of $2,800. Rental payments under a_2 and a_3 increase with units sold but are fixed under a_1.

Despite the high probability of selling only 30 units, Emma still prefers to take action a_1, which is to pay a fixed fee of $2,000. That's because the high risk of low operating income (the 60% probability of selling only 30 units) is more than offset by the high return from selling 60 units, which has a 40% probability. If Emma were more averse to risk (measured in our example by the difference between operating incomes when 30 versus 60 units are sold), she might have preferred action a_2 or a_3. For example, action a_2 ensures an operating income of at least $700, greater than the operating income of $400 that she would earn under action a_1 if only 30 units were sold. Of course, choosing a_2 limits the upside potential to $2,200 relative to $2,800 under a_1, if 60 units are sold. If Emma is very concerned about downside risk, however, she may be willing to forgo some upside benefits to protect against a $400 outcome by choosing a_2.[3]

Good Decisions and Good Outcomes

Always distinguish between a good decision and a good outcome. One can exist without the other. Suppose you are offered a one-time-only gamble tossing a coin. You will win $20 if the event is heads, but you will lose $1 if the event is tails. As a decision maker, you proceed through the logical phases: gathering information, assessing outcomes, and making a choice. You accept the bet. Why? Because the expected value is $9.50 [0.5($20) + 0.5(−$1)]. The coin is tossed and the event is tails. You lose. From your viewpoint, this was a good decision but a bad outcome.

A decision can be made only on the basis of information that is available at the time of evaluating and making the decision. By definition, uncertainty rules out guaranteeing that the best outcome will always be obtained. As in our example, it is possible that bad luck will produce bad outcomes even when good decisions have been made. A bad outcome does not mean a bad decision was made. The best protection against a bad outcome is a good decision.

Terms to Learn

This chapter and the Glossary at the end of the book contain definitions of the following important terms:

breakeven point (BEP) **(p. 68)**
choice criterion **(p. 84)**
contribution income statement **(p. 65)**
contribution margin **(p. 64)**
contribution margin per unit **(p. 65)**
contribution margin percentage **(p. 65)**
contribution margin ratio **(p. 65)**
cost-volume-profit (CVP) analysis **(p. 63)**
decision table **(p. 85)**
degree of operating leverage **(p. 76)**
event **(p. 84)**
expected monetary value **(p. 85)**
expected value **(p. 85)**
gross margin percentage **(p. 82)**
margin of safety **(p. 74)**
net income **(p. 70)**
operating leverage **(p. 76)**
outcomes **(p. 85)**
probability **(p. 84)**
probability distribution **(p. 84)**
PV graph **(p. 70)**
revenue driver **(p. 68)**
sales mix **(p. 77)**
sensitivity analysis **(p. 73)**
uncertainty **(p. 75)**

[3] For more formal approaches, refer to Moore, J. and L. Weatherford. 2001. *Decision modeling with Microsoft Excel*, 6th ed. Upper Saddle River, NJ: Prentice Hall.

Assignment Material

Note: To underscore the basic CVP relationships, the assignment material ignores income taxes unless stated otherwise.

Questions

MyAccountingLab

3-1 Define cost-volume-profit analysis.

3-2 Describe the assumptions underlying CVP analysis.

3-3 Distinguish between operating income and net income.

3-4 Define contribution margin, contribution margin per unit, and contribution margin percentage.

3-5 Describe three methods that can be used to express CVP relationships.

3-6 Why is it more accurate to describe the subject matter of this chapter as CVP analysis rather than as breakeven analysis?

3-7 "CVP analysis is both simple and simplistic. If you want realistic analysis to underpin your decisions, look beyond CVP analysis." Do you agree? Explain.

3-8 How does an increase in the income tax rate affect the breakeven point?

3-9 Describe sensitivity analysis. How has the advent of the electronic spreadsheet affected the use of sensitivity analysis?

3-10 Give an example of how a manager can decrease variable costs while increasing fixed costs.

3-11 Give an example of how a manager can increase variable costs while decreasing fixed costs.

3-12 What is operating leverage? How is knowing the degree of operating leverage helpful to managers?

3-13 "There is no such thing as a fixed cost. All costs can be 'unfixed' given sufficient time." Do you agree? What is the implication of your answer for CVP analysis?

3-14 How can a company with multiple products compute its breakeven point?

3-15 "In CVP analysis, gross margin is a less-useful concept than contribution margin." Do you agree? Explain briefly.

Exercises

MyAccountingLab

3-16 CVP computations. Fill in the blanks for each of the following independent cases.

Case	Revenues	Variable Costs	Fixed Costs	Total Costs	Operating Income	Contribution Margin Percentage
a.		$500		$ 800	$1,200	
b.	$2,000		$300		$ 200	
c.	$1,000	$700		$1,000		
d.	$1,500		$300			40%

3-17 CVP computations. Garrett Manufacturing sold 410,000 units of its product for $68 per unit in 2011. Variable cost per unit is $60 and total fixed costs are $1,640,000.

Required

1. Calculate (a) contribution margin and (b) operating income.
2. Garrett's current manufacturing process is labor intensive. Kate Schoenen, Garrett's production manager, has proposed investing in state-of-the-art manufacturing equipment, which will increase the annual fixed costs to $5,330,000. The variable costs are expected to decrease to $54 per unit. Garrett expects to maintain the same sales volume and selling price next year. How would acceptance of Schoenen's proposal affect your answers to (a) and (b) in requirement 1?
3. Should Garrett accept Schoenen's proposal? Explain.

3-18 CVP analysis, changing revenues and costs. Sunny Spot Travel Agency specializes in flights between Toronto and Jamaica. It books passengers on Canadian Air. Sunny Spot's fixed costs are $23,500 per month. Canadian Air charges passengers $1,500 per round-trip ticket.

Calculate the number of tickets Sunny Spot must sell each month to (a) break even and (b) make a target operating income of $17,000 per month in each of the following independent cases.

Required

1. Sunny Spot's variable costs are $43 per ticket. Canadian Air pays Sunny Spot 6% commission on ticket price.
2. Sunny Spot's variable costs are $40 per ticket. Canadian Air pays Sunny Spot 6% commission on ticket price.
3. Sunny Spot's variable costs are $40 per ticket. Canadian Air pays $60 fixed commission per ticket to Sunny Spot. Comment on the results.
4. Sunny Spot's variable costs are $40 per ticket. It receives $60 commission per ticket from Canadian Air. It charges its customers a delivery fee of $5 per ticket. Comment on the results.

3-19 CVP exercises. The Super Donut owns and operates six doughnut outlets in and round Kansas City. You are given the following corporate budget data for next year:

Revenues	$10,000,000
Fixed costs	$ 1,800,000
Variable costs	$ 8,000,000

Variable costs change with respect to the number of doughnuts sold.

Required Compute the budgeted operating income for each of the following deviations from the original budget data. (Consider each case independently.)

1. A 10% increase in contribution margin, holding revenues constant
2. A 10% decrease in contribution margin, holding revenues constant
3. A 5% increase in fixed costs
4. A 5% decrease in fixed costs
5. An 8% increase in units sold
6. An 8% decrease in units sold
7. A 10% increase in fixed costs and a 10% increase in units sold
8. A 5% increase in fixed costs and a 5% decrease in variable costs

3-20 CVP exercises. The Doral Company manufactures and sells pens. Currently, 5,000,000 units are sold per year at $0.50 per unit. Fixed costs are $900,000 per year. Variable costs are $0.30 per unit.

Required Consider each case separately:

1a. What is the current annual operating income?

b. What is the present breakeven point in revenues?

Compute the new operating income for each of the following changes:

2. A $0.04 per unit increase in variable costs
3. A 10% increase in fixed costs and a 10% increase in units sold
4. A 20% decrease in fixed costs, a 20% decrease in selling price, a 10% decrease in variable cost per unit, and a 40% increase in units sold

Compute the new breakeven point in units for each of the following changes:

5. A 10% increase in fixed costs
6. A 10% increase in selling price and a $20,000 increase in fixed costs

3-21 CVP analysis, income taxes. Brooke Motors is a small car dealership. On average, it sells a car for $27,000, which it purchases from the manufacturer for $23,000. Each month, Brooke Motors pays $48,200 in rent and utilities and $68,000 for salespeople's salaries. In addition to their salaries, salespeople are paid a commission of $600 for each car they sell. Brooke Motors also spends $13,000 each month for local advertisements. Its tax rate is 40%.

Required

1. How many cars must Brooke Motors sell each month to break even?
2. Brooke Motors has a target monthly net income of $51,000. What is its target monthly operating income? How many cars must be sold each month to reach the target monthly net income of $51,000?

3-22 CVP analysis, income taxes. The Express Banquet has two restaurants that are open 24-hours a day. Fixed costs for the two restaurants together total $459,000 per year. Service varies from a cup of coffee to full meals. The average sales check per customer is $8.50. The average cost of food and other variable costs for each customer is $3.40. The income tax rate is 30%. Target net income is $107,100.

Required

1. Compute the revenues needed to earn the target net income.
2. How many customers are needed to break even? To earn net income of $107,100?
3. Compute net income if the number of customers is 170,000.

3-23 CVP analysis, sensitivity analysis. Hoot Washington is the newly elected leader of the Republican Party. Media Publishers is negotiating to publish Hoot's Manifesto, a new book that promises to be an instant best-seller. The fixed costs of producing and marketing the book will be $500,000. The variable costs of producing and marketing will be $4.00 per copy sold. These costs are before any payments to Hoot. Hoot negotiates an up-front payment of $3 million, plus a 15% royalty rate on the net sales price of each book. The net sales price is the listed bookstore price of $30, minus the margin paid to the bookstore to sell the book. The normal bookstore margin of 30% of the listed bookstore price is expected to apply.

Required

1. Prepare a PV graph for Media Publishers.
2. How many copies must Media Publishers sell to (a) break even and (b) earn a target operating income of $2 million?
3. Examine the sensitivity of the breakeven point to the following changes:

   a. Decreasing the normal bookstore margin to 20% of the listed bookstore price of $30

   b. Increasing the listed bookstore price to $40 while keeping the bookstore margin at 30%

   c. Comment on the results

3-24 **CVP analysis, margin of safety.** Suppose Doral Corp.'s breakeven point is revenues of $1,100,000. Fixed costs are $660,000.

Required

1. Compute the contribution margin percentage.
2. Compute the selling price if variable costs are $16 per unit.
3. Suppose 95,000 units are sold. Compute the margin of safety in units and dollars.

3-25 **Operating leverage.** Color Rugs is holding a two-week carpet sale at Jerry's Club, a local warehouse store. Color Rugs plans to sell carpets for $500 each. The company will purchase the carpets from a local distributor for $350 each, with the privilege of returning any unsold units for a full refund. Jerry's Club has offered Color Rugs two payment alternatives for the use of space.

- Option 1: A fixed payment of $5,000 for the sale period
- Option 2: 10% of total revenues earned during the sale period

Assume Color Rugs will incur no other costs.

Required

1. Calculate the breakeven point in units for (a) option 1 and (b) option 2.
2. At what level of revenues will Color Rugs earn the same operating income under either option?
   a. For what range of unit sales will Color Rugs prefer option 1?
   b. For what range of unit sales will Color Rugs prefer option 2?
3. Calculate the degree of operating leverage at sales of 100 units for the two rental options.
4. Briefly explain and interpret your answer to requirement 3.

3-26 **CVP analysis, international cost structure differences.** Global Textiles, Inc., is considering three possible countries for the sole manufacturing site of its newest area rug: Singapore, Brazil, and the United States. All area rugs are to be sold to retail outlets in the United States for $250 per unit. These retail outlets add their own markup when selling to final customers. Fixed costs and variable cost per unit (area rug) differ in the three countries.

Country	Sales Price to Retail Outlets	Annual Fixed Costs	Variable Manufacturing Cost per Area Rug	Variable Marketing & Distribution Cost per Area Rug
Singapore	$250.00	$ 9,000,000	$75.00	$25.00
Brazil	250.00	8,400,000	60.00	15.00
United States	250.00	12,400,000	82.50	12.50

Required

1. Compute the breakeven point for Global Textiles, Inc., in each country in (a) units sold and (b) revenues.
2. If Global Textiles, Inc., plans to produce and sell 75,000 rugs in 2011, what is the budgeted operating income for each of the three manufacturing locations? Comment on the results.

3-27 **Sales mix, new and upgrade customers.** Data 1-2-3 is a top-selling electronic spreadsheet product. Data is about to release version 5.0. It divides its customers into two groups: new customers and upgrade customers (those who previously purchased Data 1-2-3, 4.0 or earlier versions). Although the same physical product is provided to each customer group, sizable differences exist in selling prices and variable marketing costs:

	New Customers		Upgrade Customers	
Selling price		$275		$100
Variable costs				
Manufacturing	$35		$35	
Marketing	65	100	15	50
Contribution margin		$175		$ 50

The fixed costs of Data 1-2-3, 5.0 are $15,000,000. The planned sales mix in units is 60% new customers and 40% upgrade customers.

Required

1. What is the Data 1-2-3, 5.0 breakeven point in units, assuming that the planned 60%:40% sales mix is attained?
2. If the sales mix is attained, what is the operating income when 220,000 total units are sold?
3. Show how the breakeven point in units changes with the following customer mixes:
   a. New 40% and Upgrade 60%
   b. New 80% and Upgrade 20%
   c. Comment on the results

3-28 Sales mix, three products. Bobbie's Bagel Shop sells only coffee and bagels. Bobbie estimates that every time she sells one bagel, she sells four cups of coffee. The budgeted cost information for Bobbie's products for 2011 follows:

	Coffee		Bagels
Selling Price	$2.50		$3.75
Product ingredients	$0.25		$0.50
Hourly sales staff (cost per unit)	$0.50		$1.00
Packaging	$0.50		$0.25
Fixed Costs			
Rent on store and equipment		$5,000	
Marketing and advertising cost		$2,000	

Required

1. How many cups of coffee and how many bagels must Bobbie sell in order to break even assuming the sales mix of four cups of coffee to one bagel, given previously?
2. If the sales mix is four cups of coffee to one bagel, how many units of each product does Bobbie need to sell to earn operating income before tax of $28,000?
3. Assume that Bobbie decides to add the sale of muffins to her product mix. The selling price for muffins is $3.00 and the related variable costs are $0.75. Assuming a sales mix of three cups of coffee to two bagels to one muffin, how many units of each product does Bobbie need to sell in order to break even? Comment on the results.

3-29 CVP, Not for profit. Monroe Classical Music Society is a not-for-profit organization that brings guest artists to the community's greater metropolitan area. The Music Society just bought a small concert hall in the center of town to house its performances. The mortgage payments on the concert hall are expected to be $2,000 per month. The organization pays its guest performers $1,000 per concert and anticipates corresponding ticket sales to be $2,500 per event. The Music Society also incurs costs of approximately $500 per concert for marketing and advertising. The organization pays its artistic director $50,000 per year and expects to receive $40,000 in donations in addition to its ticket sales.

Required

1. If the Monroe Classical Music Society just breaks even, how many concerts does it hold?
2. In addition to the organization's artistic director, the Music Society would like to hire a marketing director for $40,000 per year. What is the breakeven point? The Music Society anticipates that the addition of a marketing director would allow the organization to increase the number of concerts to 60 per year. What is the Music Society's operating income/(loss) if it hires the new marketing director?
3. The Music Society expects to receive a grant that would provide the organization with an additional $20,000 toward the payment of the marketing director's salary. What is the breakeven point if the Music Society hires the marketing director and receives the grant?

3-30 Contribution margin, decision making. Lurvey Men's Clothing's revenues and cost data for 2011 are as follows:

Revenues		$600,000
Cost of goods sold		300,000
Gross margin		300,000
Operating costs:		
Salaries fixed	$170,000	
Sales commissions (10% of sales)	60,000	
Depreciation of equipment and fixtures	20,000	
Store rent ($4,500 per month)	54,000	
Other operating costs	45,000	349,000
Operating income (loss)		$ (49,000)

Mr. Lurvey, the owner of the store, is unhappy with the operating results. An analysis of other operating costs reveals that it includes $30,000 variable costs, which vary with sales volume, and $15,000 (fixed) costs.

Required

1. Compute the contribution margin of Lurvey Men's Clothing.
2. Compute the contribution margin percentage.
3. Mr. Lurvey estimates that he can increase revenues by 15% by incurring additional advertising costs of $13,000. Calculate the impact of the additional advertising costs on operating income.

3-31 Contribution margin, gross margin, and margin of safety. Mirabella Cosmetics manufactures and sells a face cream to small ethnic stores in the greater New York area. It presents the monthly operating income statement shown here to George Lopez, a potential investor in the business. Help Mr. Lopez understand Mirabella's cost structure.

	A	B	C	D
1	**Mirabella Cosmetics**			
2	**Operating Income Statement, June 2011**			
3	Units sold			10,000
4	Revenues			$100,000
5	Cost of goods sold			
6	Variable manufacturing costs		$55,000	
7	Fixed manufacturing costs		20,000	
8	Total			75,000
9	Gross margin			25,000
10	Operating costs			
11	Variable marketing costs		$ 5,000	
12	Fixed marketing & administration costs		10,000	
13	Total operating costs			15,000
14	Operating income			$ 10,000

Required

1. Recast the income statement to emphasize contribution margin.
2. Calculate the contribution margin percentage and breakeven point in units and revenues for June 2011.
3. What is the margin of safety (in units) for June 2011?
4. If sales in June were only 8,000 units and Mirabella's tax rate is 30%, calculate its net income.

3-32 Uncertainty and expected costs. Foodmart Corp, an international retail giant, is considering implementing a new business to business (B2B) information system for processing purchase orders. The current system costs Foodmart $2,500,000 per month and $50 per order. Foodmart has two options, a partially automated B2B and a fully automated B2B system. The partially automated B2B system will have a fixed cost of $10,000,000 per month and a variable cost of $40 per order. The fully automated B2B system has a fixed cost of $20,000,000 per month and $25 per order.

Based on data from the last two years, Foodmart has determined the following distribution on monthly orders:

Monthly Number of Orders	Probability
350,000	0.15
450,000	0.20
550,000	0.35
650,000	0.20
750,000	0.10

Required

1. Prepare a table showing the cost of each plan for each quantity of monthly orders.
2. What is the expected cost of each plan?
3. In addition to the information systems costs, what other factors should Foodmart consider before deciding to implement a new B2B system?

Problems

3-33 CVP analysis, service firm. Lifetime Escapes generates average revenue of $5,000 per person on its five-day package tours to wildlife parks in Kenya. The variable costs per person are as follows:

Airfare	$1,400
Hotel accommodations	1,100
Meals	300
Ground transportation	100
Park tickets and other costs	800
Total	$3,700

Annual fixed costs total $520,000.

Required

1. Calculate the number of package tours that must be sold to break even.
2. Calculate the revenue needed to earn a target operating income of $91,000.
3. If fixed costs increase by $32,000, what decrease in variable cost per person must be achieved to maintain the breakeven point calculated in requirement 1?

3-34 CVP, target operating income, service firm. Snow Leopard Daycare provides daycare for children Mondays through Fridays. Its monthly variable costs per child are as follows:

Lunch and snacks	$150
Educational supplies	60
Other supplies (paper products, toiletries, etc.)	20
Total	$230

Monthly fixed costs consist of the following:

Rent	$2,150
Utilities	200
Insurance	250
Salaries	2,350
Miscellaneous	650
Total	$5,600

Snow Leopard charges each parent $580 per child.

Required

1. Calculate the breakeven point.
2. Snow Leopard's target operating income is $10,500 per month. Compute the number of children who must be enrolled to achieve the target operating income.
3. Snow Leopard lost its lease and had to move to another building. Monthly rent for the new building is $3,150. At the suggestion of parents, Snow Leopard plans to take children on field trips. Monthly costs of the field trips are $1,300. By how much should Snow Leopard increase fees per child to meet the target operating income of $10,500 per month, assuming the same number of children as in requirement 2?

3-35 CVP analysis, margin of safety. (CMA, adapted) Technology Solutions sells a ready-to-use software product for small businesses. The current selling price is $300. Projected operating income for 2011 is $490,000 based on a sales volume of 10,000 units. Variable costs of producing the software are $120 per unit sold plus an additional cost of $5 per unit for shipping and handling. Technology Solutions annual fixed costs are $1,260,000.

Required

1. Calculate Technology Solutions breakeven point and margin of safety in units.
2. Calculate the company's operating income for 2011 if there is a 10% increase in unit sales.
3. For 2012, management expects that the per unit production cost of the software will increase by 30%, but the shipping and handling costs per unit will decrease by 20%. Calculate the sales revenue Technology Solutions must generate for 2012 to maintain the current year's operating income if the selling price remains unchanged, assuming all other data as in the original problem.

3-36 CVP analysis, income taxes. (CMA, adapted) R. A. Ro and Company, a manufacturer of quality handmade walnut bowls, has had a steady growth in sales for the past five years. However, increased competition has led Mr. Ro, the president, to believe that an aggressive marketing campaign will be necessary next year to maintain the company's present growth. To prepare for next year's marketing campaign, the company's controller has prepared and presented Mr. Ro with the following data for the current year, 2011:

Variable cost (per bowl)	
Direct materials	$ 3.25
Direct manufacturing labor	8.00
Variable overhead (manufacturing, marketing, distribution, and customer service)	2.50
Total variable cost per bowl	$ 13.75
Fixed costs	
Manufacturing	$ 25,000
Marketing, distribution, and customer service	110,000
Total fixed costs	$135,000
Selling price	25.00
Expected sales, 20,000 units	$500,000
Income tax rate	40%

Required

1. What is the projected net income for 2011?
2. What is the breakeven point in units for 2011?
3. Mr. Ro has set the revenue target for 2012 at a level of $550,000 (or 22,000 bowls). He believes an additional marketing cost of $11,250 for advertising in 2012, with all other costs remaining constant, will be necessary to attain the revenue target. What is the net income for 2012 if the additional $11,250 is spent and the revenue target is met?
4. What is the breakeven point in revenues for 2012 if the additional $11,250 is spent for advertising?
5. If the additional $11,250 is spent, what are the required 2012 revenues for 2012 net income to equal 2011 net income?
6. At a sales level of 22,000 units, what maximum amount can be spent on advertising if a 2012 net income of $60,000 is desired?

3-37 CVP, sensitivity analysis. The Brown Shoe Company produces its famous shoe, the Divine Loafer that sells for $60 per pair. Operating income for 2011 is as follows:

Sales revenue ($60 per pair)	$300,000
Variable cost ($25 per pair)	125,000
Contribution margin	175,000
Fixed cost	100,000
Operating income	$ 75,000

Brown Shoe Company would like to increase its profitability over the next year by at least 25%. To do so, the company is considering the following options:

Required

1. Replace a portion of its variable labor with an automated machining process. This would result in a 20% decrease in variable cost per unit, but a 15% increase in fixed costs. Sales would remain the same.
2. Spend $30,000 on a new advertising campaign, which would increase sales by 20%.
3. Increase both selling price by $10 per unit and variable costs by $7 per unit by using a higher quality leather material in the production of its shoes. The higher priced shoe would cause demand to drop by approximately 10%.
4. Add a second manufacturing facility which would double Brown's fixed costs, but would increase sales by 60%.

Evaluate each of the alternatives considered by Brown Shoes. Do any of the options meet or exceed Brown's targeted increase in income of 25%? What should Brown do?

3-38 CVP analysis, shoe stores. The WalkRite Shoe Company operates a chain of shoe stores that sell 10 different styles of inexpensive men's shoes with identical unit costs and selling prices. A unit is defined as a pair of shoes. Each store has a store manager who is paid a fixed salary. Individual salespeople receive a fixed salary and a sales commission. WalkRite is considering opening another store that is expected to have the revenue and cost relationships shown here:

Home Insert Page Layout Formulas Data Review View

	A	B	C	D	E
1	**Unit Variable Data (per pair of shoes)**			**Annual Fixed Costs**	
2	Selling price	$30.00		Rent	$ 60,000
3	Cost of shoes	$19.50		Salaries	200,000
4	Sales commission	1.50		Advertising	80,000
5	Variable cost per unit	$21.00		Other fixed costs	20,000
6				Total fixed costs	$360,000

Consider each question independently:

Required

1. What is the annual breakeven point in (a) units sold and (b) revenues?
2. If 35,000 units are sold, what will be the store's operating income (loss)?
3. If sales commissions are discontinued and fixed salaries are raised by a total of $81,000, what would be the annual breakeven point in (a) units sold and (b) revenues?
4. Refer to the original data. If, in addition to his fixed salary, the store manager is paid a commission of $0.30 per unit sold, what would be the annual breakeven point in (a) units sold and (b) revenues?
5. Refer to the original data. If, in addition to his fixed salary, the store manager is paid a commission of $0.30 *per unit in excess of the breakeven point*, what would be the store's operating income if 50,000 units were sold?

3-39 CVP analysis, shoe stores (continuation of 3-38). Refer to requirement 3 of Problem 3-38. In this problem, assume the role of the owner of WalkRite.

Required

1. Calculate the number of units sold at which the owner of WalkRite would be indifferent between the original salary-plus-commissions plan for salespeople and the higher fixed-salaries-only plan.
2. As owner, which sales compensation plan would you choose if forecasted annual sales of the new store were at least 55,000 units? What do you think of the motivational aspect of your chosen compensation plan?
3. Suppose the target operating income is $168,000. How many units must be sold to reach the target operating income under (a) the original salary-plus-commissions plan and (b) the higher-fixed-salaries-only plan?
4. You open the new store on January 1, 2011, with the original salary-plus-commission compensation plan in place. Because you expect the cost of the shoes to rise due to inflation, you place a firm bulk order for 50,000 shoes and lock in the $19.50 price per unit. But, toward the end of the year, only 48,000 shoes are sold, and you authorize a markdown of the remaining inventory to $18 per unit. Finally, all units are sold. Salespeople, as usual, get paid a commission of 5% of revenues. What is the annual operating income for the store?

3-40 Alternate cost structures, uncertainty, and sensitivity analysis. Stylewise Printing Company currently leases its only copy machine for $1,000 a month. The company is considering replacing this leasing agreement with a new contract that is entirely commission based. Under the new agreement Stylewise would pay a commission for its printing at a rate of $10 for every 500 pages printed. The company currently charges $0.15 per page to its customers. The paper used in printing costs the company $.03 per page and other variable costs, including hourly labor amount to $.04 per page.

Required

1. What is the company's breakeven point under the current leasing agreement? What is it under the new commission based agreement?
2. For what range of sales levels will Stylewise prefer (a) the fixed lease agreement (b) the commission agreement?
3. Do this question only if you have covered the chapter appendix in your class. Stylewise estimates that the company is equally likely to sell 20,000; 40,000; 60,000; 80,000; or 100,000 pages of print. Using information from the original problem, prepare a table that shows the expected profit at each sales level under the fixed leasing agreement and under the commission based agreement. What is the expected value of each agreement? Which agreement should Stylewise choose?

3-41 CVP, alternative cost structures. PC Planet has just opened its doors. The new retail store sells refurbished computers at a significant discount from market prices. The computers cost PC Planet $100 to purchase and require 10 hours of labor at $15 per hour. Additional variable costs, including wages for sales personnel, are $50 per computer. The newly refurbished computers are resold to customers for $500. Rent on the retail store costs the company $4,000 per month.

Required

1. How many computers does PC Planet have to sell each month to break even?
2. If PC Planet wants to earn $5,000 per month after all expenses, how many computers does the company need to sell?
3. PC Planet can purchase already refurbished computers for $200. This would mean that all labor required to refurbish the computers could be eliminated. What would PC Planet's new breakeven point be if it decided to purchase the computers already refurbished?
4. Instead of paying the monthly rental fee for the retail space, PC Planet has the option of paying its landlord a 20% commission on sales. Assuming the original facts in the problem, at what sales level would PC Planet be indifferent between paying a fixed amount of monthly rent and paying a 20% commission on sales?

3-42 CVP analysis, income taxes, sensitivity. (CMA, adapted) Agro Engine Company manufactures and sells diesel engines for use in small farming equipment. For its 2012 budget, Agro Engine Company estimates the following:

Selling price	$ 3,000
Variable cost per engine	$ 500
Annual fixed costs	$3,000,000
Net income	$1,500,000
Income tax rate	25%

The first quarter income statement, as of March 31, reported that sales were not meeting expectations. During the first quarter, only 300 units had been sold at the current price of $3,000. The income statement showed that variable and fixed costs were as planned, which meant that the 2012 annual net income projection would not be met unless management took action. A management committee was formed and presented the following mutually exclusive alternatives to the president:

a. Reduce the selling price by 20%. The sales organization forecasts that at this significantly reduced price, 2,000 units can be sold during the remainder of the year. Total fixed costs and variable cost per unit will stay as budgeted.
b. Lower variable cost per unit by $50 through the use of less-expensive direct materials. The selling price will also be reduced by $250, and sales of 1,800 units are expected for the remainder of the year.
c. Reduce fixed costs by 20% and lower the selling price by 10%. Variable cost per unit will be unchanged. Sales of 1,700 units are expected for the remainder of the year.

Required

1. If no changes are made to the selling price or cost structure, determine the number of units that Agro Engine Company must sell (a) to break even and (b) to achieve its net income objective.
2. Determine which alternative Agro Engine should select to achieve its net income objective. Show your calculations.

3-43 Choosing between compensation plans, operating leverage. (CMA, adapted) Marston Corporation manufactures pharmaceutical products that are sold through a network of external sales agents. The agents are paid a commission of 18% of revenues. Marston is considering replacing the sales agents with its own salespeople, who would be paid a commission of 10% of revenues and total salaries of $2,080,000. The income statement for the year ending December 31, 2011, under the two scenarios is shown here.

	A	B	C	D	E
1	Marston Corporation				
2	Income Statement				
3	For theYear Ended December 31, 2011				
4		Using Sales Agents		Using Own Sales Force	
5	Revenues		$26,000,000		$26,000,000
6	Cost of goods sold				
7	Variable	$11,700,000		$11,700,000	
8	Fixed	2,870,000	14,570,000	2,870,000	14,570,000
9	Gross margin		11,430,000		11,430,000
10	Marketing costs				
11	Commissions	$ 4,680,000		$ 2,600,000	
12	Fixed costs	3,420,000	8,100,000	5,500,000	8,100,000
13	Operating income		$ 3,330,000		$ 3,330,000

Required

1. Calculate Marston's 2011 contribution margin percentage, breakeven revenues, and degree of operating leverage under the two scenarios.
2. Describe the advantages and disadvantages of each type of sales alternative.
3. In 2012, Marston uses its own salespeople, who demand a 15% commission. If all other cost behavior patterns are unchanged, how much revenue must the salespeople generate in order to earn the same operating income as in 2011?

3-44 Sales mix, three products. The Ronowski Company has three product lines of belts—A, B, and C—with contribution margins of $3, $2, and $1, respectively. The president foresees sales of 200,000 units in the coming period, consisting of 20,000 units of A, 100,000 units of B, and 80,000 units of C. The company's fixed costs for the period are $255,000.

Required

1. What is the company's breakeven point in units, assuming that the given sales mix is maintained?
2. If the sales mix is maintained, what is the total contribution margin when 200,000 units are sold? What is the operating income?
3. What would operating income be if 20,000 units of A, 80,000 units of B, and 100,000 units of C were sold? What is the new breakeven point in units if these relationships persist in the next period?

3-45 Multiproduct CVP and decision making. Pure Water Products produces two types of water filters. One attaches to the faucet and cleans all water that passes through the faucet. The other is a pitcher-cum-filter that only purifies water meant for drinking.

The unit that attaches to the faucet is sold for $80 and has variable costs of $20.
The pitcher-cum-filter sells for $90 and has variable costs of $25.

Pure Water sells two faucet models for every three pitchers sold. Fixed costs equal $945,000.

Required

1. What is the breakeven point in unit sales and dollars for each type of filter at the current sales mix?
2. Pure Water is considering buying new production equipment. The new equipment will increase fixed cost by $181,400 per year and will decrease the variable cost of the faucet and the pitcher units by $5 and $9 respectively. Assuming the same sales mix, how many of each type of filter does Pure Water need to sell to break even?
3. Assuming the same sales mix, at what total sales level would Pure Water be indifferent between using the old equipment and buying the new production equipment? If total sales are expected to be 30,000 units, should Pure Water buy the new production equipment?

3-46 Sales mix, two products. The Stackpole Company retails two products: a standard and a deluxe version of a luggage carrier. The budgeted income statement for next period is as follows:

	Standard Carrier	Deluxe Carrier	Total
Units sold	187,500	62,500	250,000
Revenues at $28 and $50 per unit	$5,250,000	$3,125,000	$8,375,000
Variable costs at $18 and $30 per unit	3,375,000	1,875,000	5,250,000
Contribution margins at $10 and $20 per unit	$1,875,000	$1,250,000	3,125,000
Fixed costs			2,250,000
Operating income			$ 875,000

Required

1. Compute the breakeven point in units, assuming that the planned sales mix is attained.
2. Compute the breakeven point in units (a) if only standard carriers are sold and (b) if only deluxe carriers are sold.
3. Suppose 250,000 units are sold but only 50,000 of them are deluxe. Compute the operating income. Compute the breakeven point in units. Compare your answer with the answer to requirement 1. What is the major lesson of this problem?

3-47 Gross margin and contribution margin. The Museum of America is preparing for its annual appreciation dinner for contributing members. Last year, 525 members attended the dinner. Tickets for the dinner were $24 per attendee. The profit report for last year's dinner follows.

Ticket sales	$12,600
Cost of dinner	15,300
Gross margin	(2,700)
Invitations and paperwork	2,500
Profit (loss)	$(5,200)

This year the dinner committee does not want to lose money on the dinner. To help achieve its goal, the committee analyzed last year's costs. Of the $15,300 cost of the dinner, $9,000 were fixed costs and $6,300 were variable costs. Of the $2,500 cost of invitations and paperwork, $1,975 were fixed and $525 were variable.

Required

1. Prepare last year's profit report using the contribution margin format.
2. The committee is considering expanding this year's dinner invitation list to include volunteer members (in addition to contributing members). If the committee expands the dinner invitation list, it expects attendance to double. Calculate the effect this will have on the profitability of the dinner assuming fixed costs will be the same as last year.

3-48 Ethics, CVP analysis. Allen Corporation produces a molded plastic casing, LX201, for desktop computers. Summary data from its 2011 income statement are as follows:

Revenues	$5,000,000
Variable costs	3,000,000
Fixed costs	2,160,000
Operating income	$ (160,000)

Jane Woodall, Allen's president, is very concerned about Allen Corporation's poor profitability. She asks Max Lemond, production manager, and Lester Bush, controller, to see if there are ways to reduce costs.

After two weeks, Max returns with a proposal to reduce variable costs to 52% of revenues by reducing the costs Allen currently incurs for safe disposal of wasted plastic. Lester is concerned that this would expose the company to potential environmental liabilities. He tells Max, "We would need to estimate some of these potential environmental costs and include them in our analysis." "You can't do that," Max replies. "We are not violating any laws. There is some possibility that we may have to incur environmental costs in the future, but if we bring it up now, this proposal will not go through because our senior management always assumes these costs to be larger than they turn out to be. The market is very tough, and we are in danger of shutting down the company and costing all of us our jobs. The only reason our competitors are making money is because they are doing exactly what I am proposing."

Required

1. Calculate Allen Corporation's breakeven revenues for 2011.
2. Calculate Allen Corporation's breakeven revenues if variable costs are 52% of revenues.
3. Calculate Allen Corporation's operating income for 2011 if variable costs had been 52% of revenues.
4. Given Max Lemond's comments, what should Lester Bush do?

Collaborative Learning Problem

3-49 Deciding where to produce. (CMA, adapted) The Domestic Engines Co. produces the same power generators in two Illinois plants, a new plant in Peoria and an older plant in Moline. The following data are available for the two plants:

	A	B	C	D	E
1		Peoria		Moline	
2	Selling price		$150.00		$150.00
3	Variable manufacturing cost per unit	$72.00		$88.00	
4	Fixed manufacturing cost per unit	30.00		15.00	
5	Variable marketing and distribution cost per unit	14.00		14.00	
6	Fixed marketing and distribution cost per unit	19.00		14.50	
7	Total cost per unit		135.00		131.50
8	Operating income per unit		$ 15.00		$ 18.50
9	Production rate per day	400	units	320	units
10	Normal annual capacity usage	240	days	240	days
11	Maximum annual capacity	300	days	300	days

All fixed costs per unit are calculated based on a normal capacity usage consisting of 240 working days. When the number of working days exceeds 240, overtime charges raise the variable manufacturing costs of additional units by $3.00 per unit in Peoria and $8.00 per unit in Moline.

Domestic Engines Co. is expected to produce and sell 192,000 power generators during the coming year. Wanting to take advantage of the higher operating income per unit at Moline, the company's production manager has decided to manufacture 96,000 units at each plant, resulting in a plan in which Moline operates at capacity (320 units per day $\times$ 300 days) and Peoria operates at its normal volume (400 units per day $\times$ 240 days).

Required

1. Calculate the breakeven point in units for the Peoria plant and for the Moline plant.
2. Calculate the operating income that would result from the production manager's plan to produce 96,000 units at each plant.
3. Determine how the production of 192,000 units should be allocated between the Peoria and Moline plants to maximize operating income for Domestic Engines Co. Show your calculations.